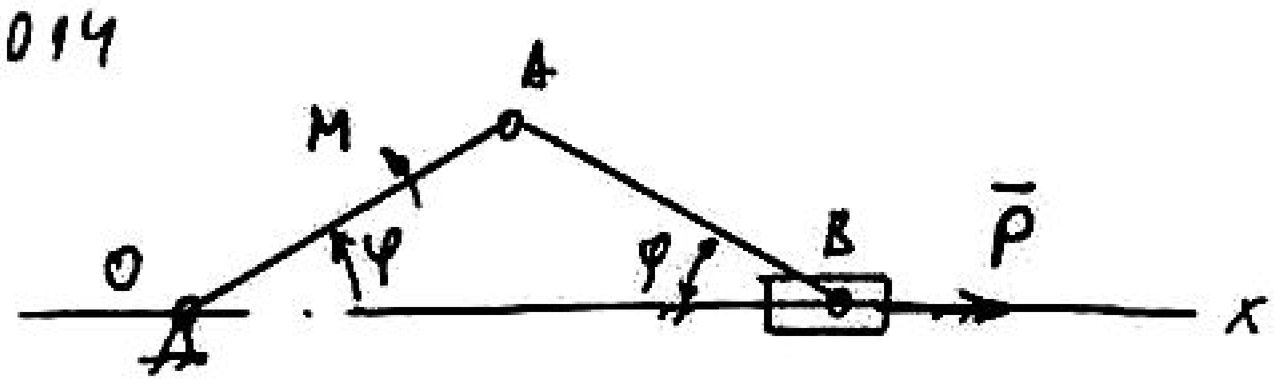
1

ДV

014



Дано: $OA = AB = l$, φ, P, горизонтальная плоскость, равновесие

M – ?

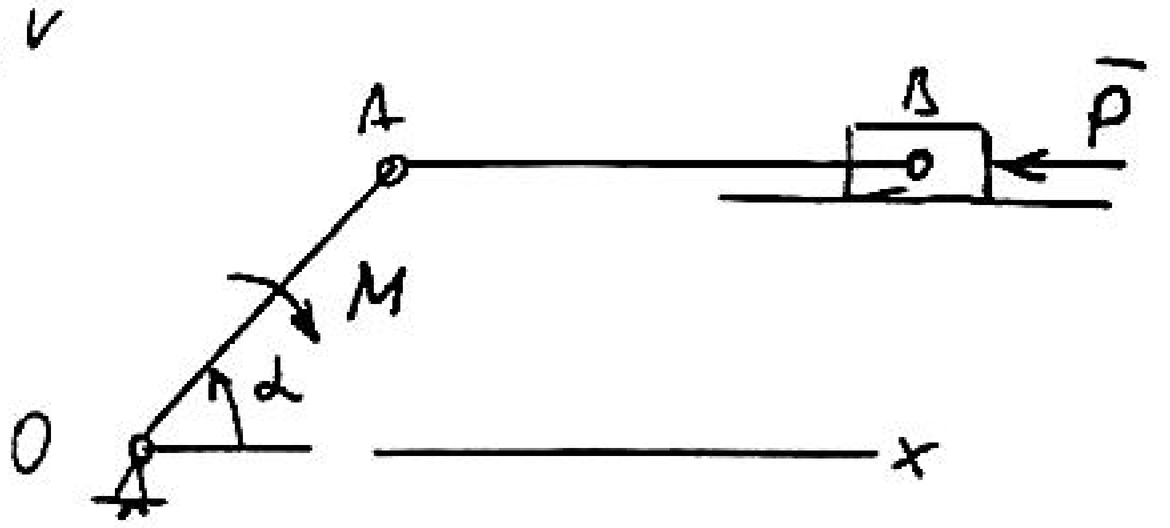
$\boxed{\text{Принцип возможных перемещений}}$

$\sum_k \bar F_k \delta \bar r_k = 0$ $\qquad M\delta\varphi + P\delta x_B = 0$

$x_B = 2l\cos\varphi$, $\quad \delta x_B = \frac{\partial x_B}{\partial \varphi}\delta\varphi = -2l\sin\varphi\,\delta\varphi$

$M\cancel{\delta\varphi} - P\cdot 2l\sin\varphi\,\cancel{\delta\varphi} = 0$ $\qquad \boxed{M = 2Pl\sin\varphi}$

ДV



Дано: $OA = l$, $AB = 2l$, $\alpha = 45°$, P

M – ? горизонтальная плоскость, равновесие

Принцип возможных перемещений

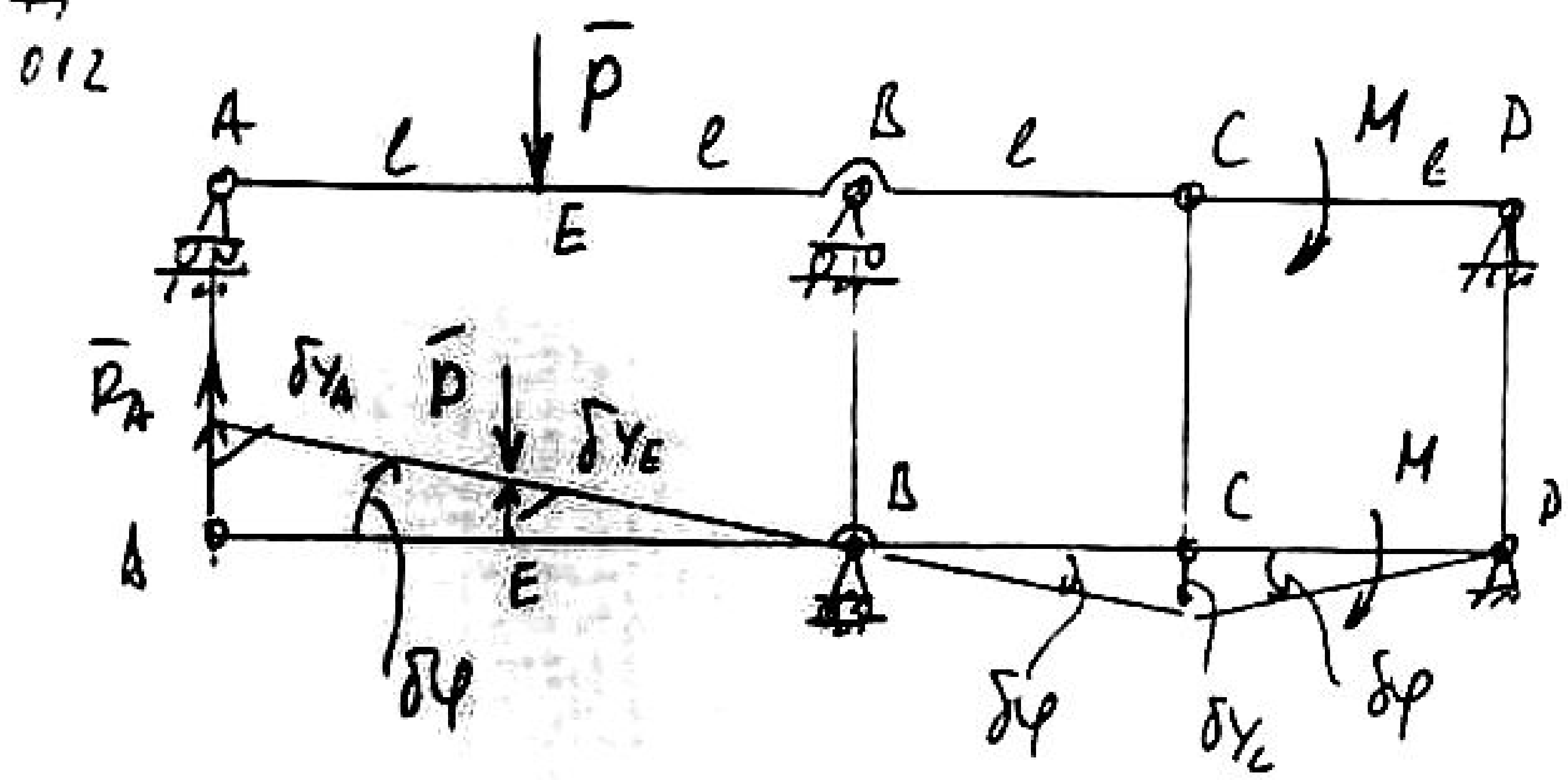
$\sum \bar F_k \delta \bar r_k = 0$ $\qquad -M\delta\alpha - P\delta x_B = 0$

$x_B = OA\cos\alpha + AB = l\cos\alpha + 2l$, $\quad \delta x_B = \frac{\partial x_B}{\partial\alpha}\cdot\delta\alpha = -l\sin\alpha\,\delta\alpha$

$-M\cancel{\delta\alpha} + Pl\sin\alpha\,\cancel{\delta\alpha} = 0$ $\qquad \boxed{M = Pl\sin\alpha = Pl/\sqrt{2}}$

ДV

012



Дано: P, l, $M = Pl$

R_A – ?

Принцип возможных перемещений

$R_A\delta y_A - P\delta y_E - M\delta\varphi = 0$

$\delta y_A = 2l\,\delta\varphi$, $\quad \delta y_E = l\,\delta\varphi$

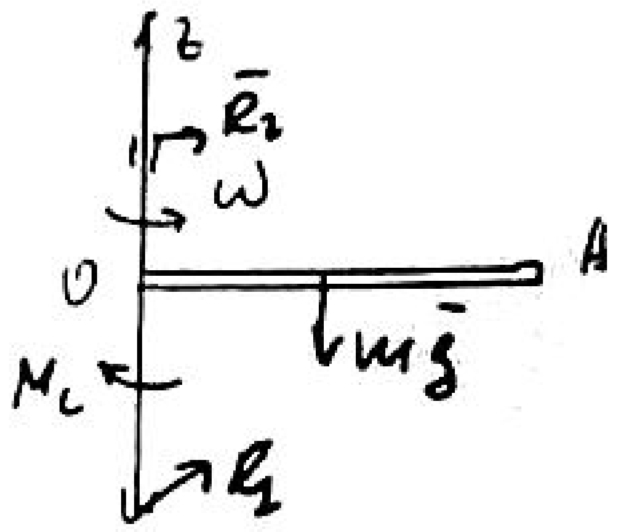
$R_A\cdot 2l\cancel{\delta\varphi} - Pl\cancel{\delta\varphi} - M\cancel{\delta\varphi} = 0$

$R_A = \frac{Pl + M}{2l} = P$

Вращательное движение

Д II

522



$OA = l$, m, ω_0, тормозит $M_c = \alpha\omega$

$\omega = \frac{\omega_0}{3}$, найти T (время)

$$J\frac{d\omega}{dt} = \Sigma M_z(\bar{F}_k^{(e)}) = -\alpha\omega$$

$$\frac{d\omega}{\omega} = -\frac{\alpha}{J}dt,$$ интегрируем

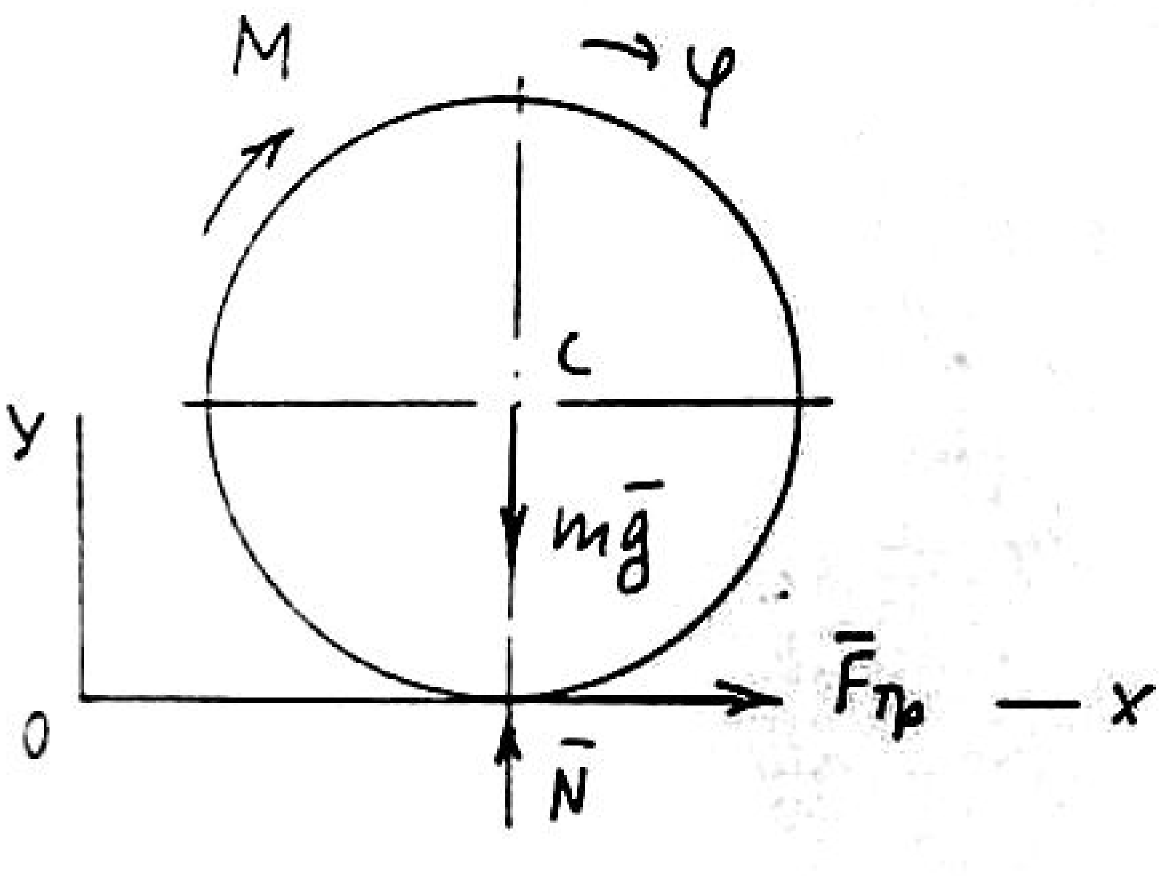
$\ln\omega = -\frac{\alpha}{J}t + C_1$, нач. условие: $t=0$ $\omega=\omega_0$ $\ln\omega_0 = C_1$

$\ln\omega = \ln\omega_0 - \frac{\alpha}{J}t$, $t = \frac{J}{\alpha}\ln\frac{\omega_0}{\omega}$; при $\omega_0 = \frac{\omega}{3}$ $t = T$

$\boxed{T = \frac{J}{\alpha}\ln 3}$ $J = \frac{ml^2}{3}$

Дифференциальные уравнения плоского движения

$$m\ddot{x}_c = \Sigma F_{kx}^{(e)}, \quad m\ddot{y}_c = \Sigma F_{ky}^{(e)}, \quad J_{cz}\,\varepsilon = \Sigma M_{cz}(\bar{F}_k^{(e)})$$



Дано: $m = 20$ кг, $M = 30$ Н·м, $R = 0{,}2$ м

катится без скольжения

Определить $F_{тр}$

Решение.

$m\ddot{x}_c = \Sigma F_{kx}^{(e)} = F_{тр}$ (1), $m\ddot{y}_c = \Sigma F_{ky}^{(e)} = N - mg = 0$ ($y_c = R$)

$J_{cz}\,\varepsilon = \Sigma M_{cz}(\bar{F}_k^{(e)}) = M - F_{тр}R$; $J_{cz} = \frac{mR^2}{2}$, $\omega = \frac{v_c}{R}$, $\varepsilon = \frac{a_c}{R} = \frac{\ddot{x}_c}{R}$

$\frac{mR^2}{2}\cdot\frac{\ddot{x}_c}{R} = M - F_{тр}R$; $m\ddot{x}_c = 2\frac{M}{R} - 2F_{тр}$ (2)

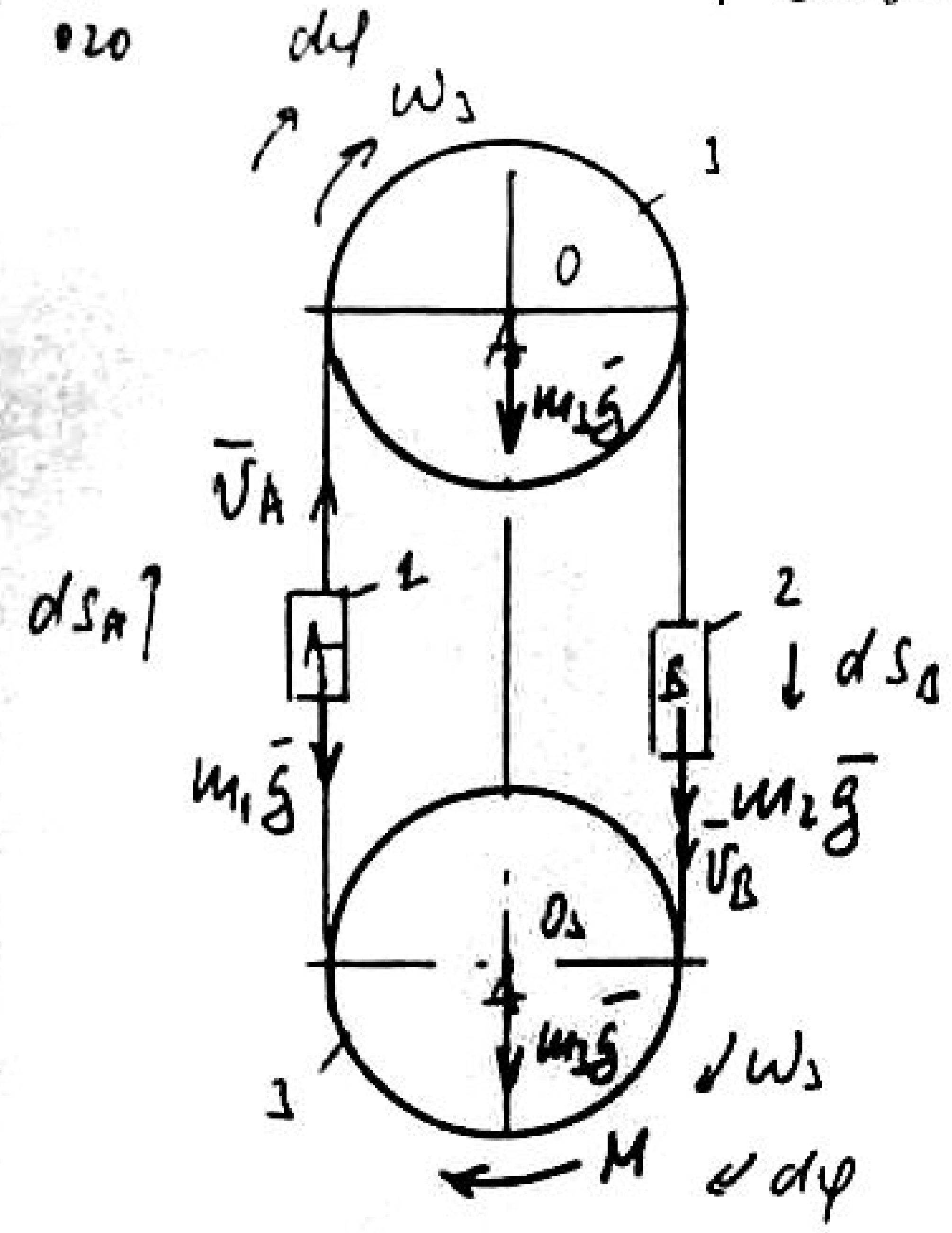
$$-\begin{cases} m\ddot{x}_c = F_{тр} \\ m\ddot{x}_c = 2\frac{M}{R} - 2F_{тр} \end{cases}$$

$$F_{тр} - 2\frac{M}{R} + 2F_{тр} = 0$$

$$F_{тр} = \frac{2}{3}\frac{M}{R} = \frac{2}{3}\cdot\frac{30}{0{,}2} = 100 \text{ Н}$$

Д IV
020

Теорема об изменении кинетической энергии в дифференциальном ...



Дано: $m_1, m_2, m_3, r, \rho, M = const$

$a_A - ?$

$dT = \sum dA(\bar{F}_k^{(e)}) + \sum dA(\bar{F}_k^{(i)})$

$T = T_1 + T_2 + 2T_3$; $\quad v_A = v_B$, $\omega_3 = \frac{v_A}{r}$, $dS_A = dS_B$, $d\varphi$

$T_1 = \frac{1}{2} m_1 v_A^2$; $T_2 = \frac{1}{2} m_2 v_B^2 = \frac{1}{2} m_2 v_A^2$

$T_3 = \frac{1}{2} J_{oz} \omega_3^2 = \frac{1}{2} m_3 \rho^2 \left(\frac{v_A}{r}\right)^2$

$T = \frac{1}{2} m_1 v_A^2 + \frac{1}{2} m_2 v_A^2 + 2 \cdot \frac{1}{2} m_3 \rho^2 \left(\frac{v_A}{r}\right)^2 = \frac{1}{2}\left[m_1 + m_2 + 2m_3\left(\frac{\rho}{r}\right)^2\right]$

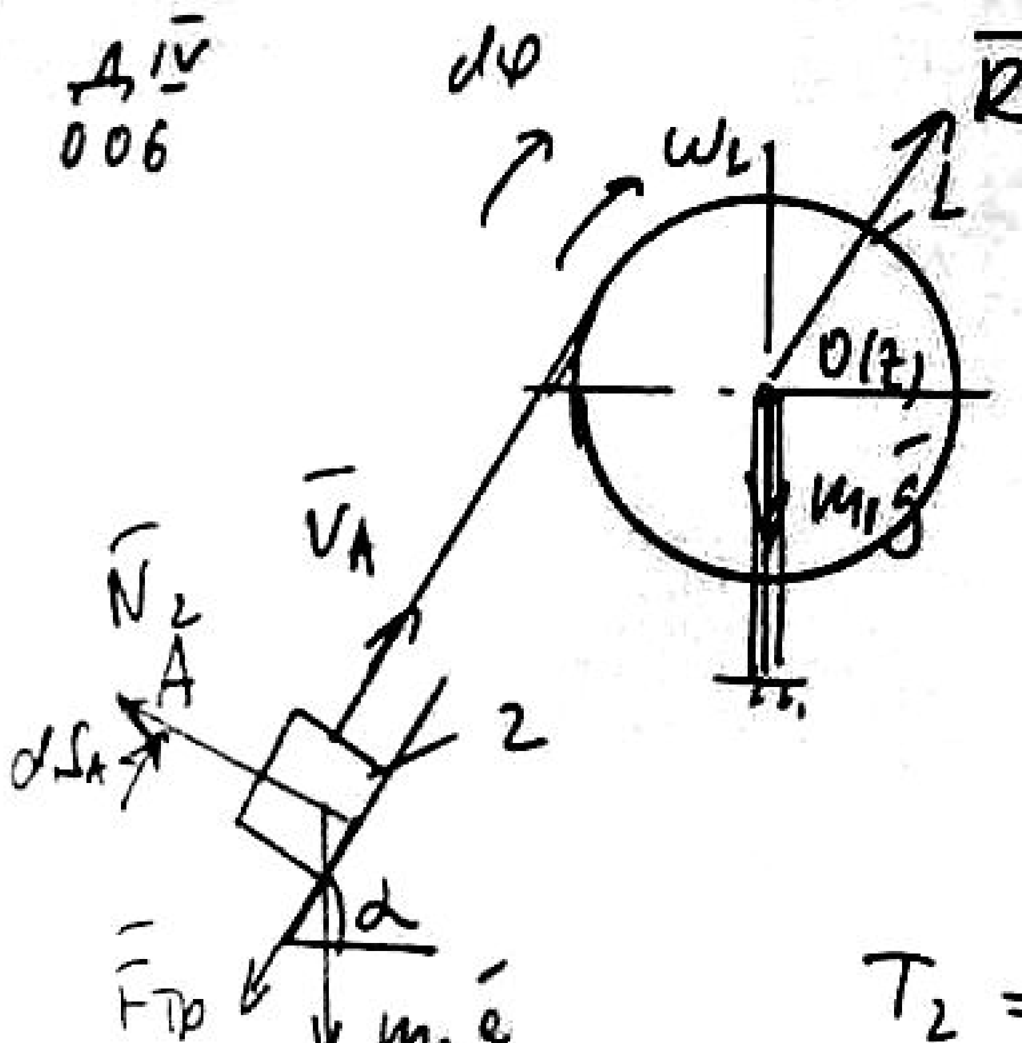
$T = \frac{1}{2}\left[m_1 + m_2 + 2m_3\left(\frac{\rho}{r}\right)^2\right] v_A^2$;

$dT = \left[m_1 + m_2 + 2m_3\left(\frac{\rho}{r}\right)^2\right] v_A \cdot dv_A = \left[m_1 + m_2 + 2m_3\left(\frac{\rho}{r}\right)^2\right] a_A \cdot dS_A$, $\quad \sum dA(\bar{F}_k^{(i)}) = 0$

$\sum dA(\bar{F}_k^{(e)}) = M d\varphi - m_1 g\, dS_A + m_2 g\, dS_A = M \frac{dS_A}{r} + (m_2 - m_1) g\, dS_A$

$\left[m_1 + m_2 + 2m_3\left(\frac{\rho}{r}\right)^2\right] a_A \cdot \cancel{dS_A} = \left[\frac{M}{r} + (m_2 - m_1) g\right] \cancel{dS_A}$; $\quad a_A = \dfrac{\frac{M}{r} + (m_2 - m_1) g}{m_1 + m_2 + 2m_3\left(\frac{\rho}{r}\right)^2}$;

Д IV
006



Дано: $m_1, r, M = const, m_2, \alpha, f$

$a_A - ?$

v_A, $\omega_1 = \frac{v_A}{r}$; dS_A, $d\varphi = \frac{dS_A}{r}$

$dT = \sum dA(\bar{F}_k^{(e)}) + \cancel{\sum dA(\bar{F}_k^{(i)})} \to 0$

$T = T_1 + T_2$, $\quad T_1 = \frac{1}{2} J_{oz} \omega_1^2 = \frac{1}{2} \frac{m_1 \cancel{r^2}}{2} \left(\frac{v_A}{\cancel{r}}\right)^2$

$T_2 = \frac{1}{2} m_2 v_A^2$, $\quad T = \frac{1}{2}\left(\frac{m_1}{2} + m_2\right) v_A^2$

$dT = \left(\frac{m_1}{2} + m_2\right) a_A \cdot dS_A$; $\quad \sum dA(\bar{F}_k^{(e)}) = M d\varphi - m_2 g \sin\alpha \cdot d$

$- F_{тр} \cdot dS_A = M \frac{dS_A}{r} - m_2 g \sin\alpha\, dS_A - f m_2 g \cos\alpha\, dS_A$

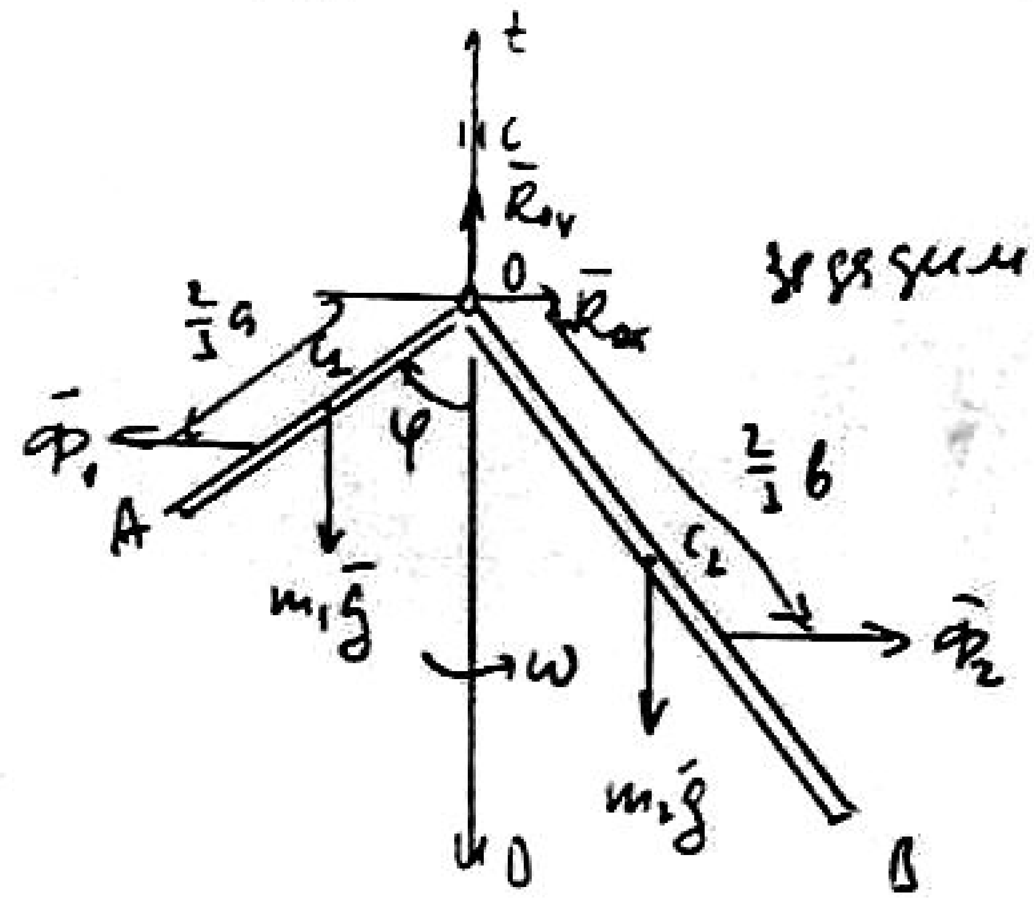
$\left(\frac{m_1}{2} + m_2\right) a_A \cancel{dS_A} = \left[\frac{M}{r} - m_2 g (\sin\alpha + f\cos\alpha)\right] \cancel{dS_A}$

Принцип Даламбера.

Д III
008

Дано: $AO = a$, $OB = b$, $\omega = const$

$\omega = \omega(\varphi)$ — ?



Звенья: стержень OA — m_1, стержень OB $m_2 = m_1 \frac{b}{a}$,

$\Phi_1 = m_1 a_{c_1}^{ц} = m_1 \omega^2 \frac{a}{2}\sin\varphi$; $\Phi_2 = m_2 a_{c_2}^{ц} = m_1 \frac{b}{a}\omega^2 \frac{b}{2}\cos\varphi$

$(m_1\bar{g}, m_2\bar{g}, \bar{R}_O, \bar{\Phi}_1, \bar{\Phi}_2) \backsim 0$

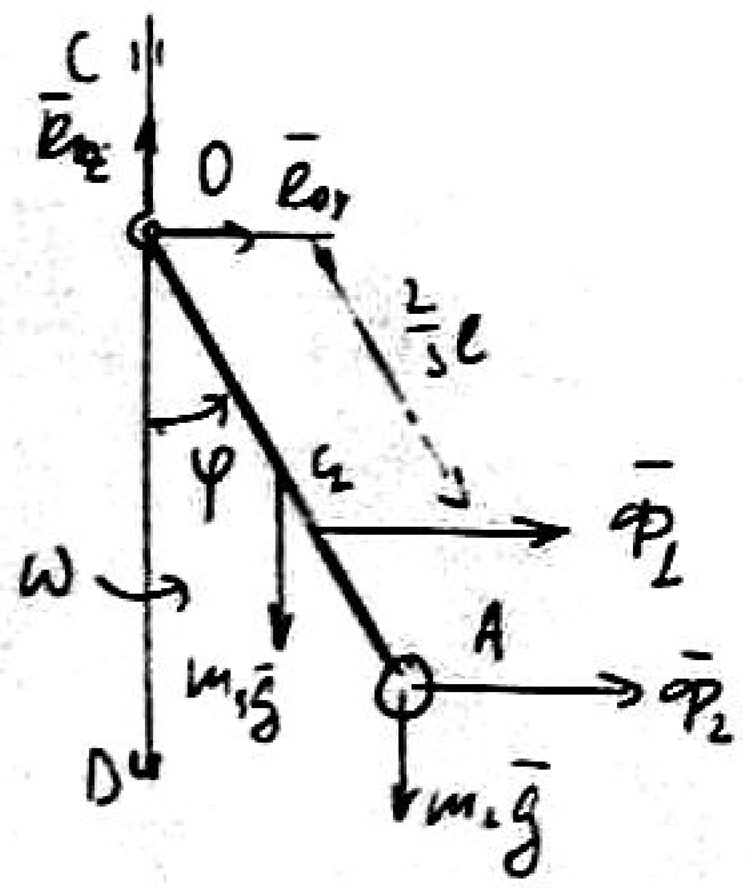
$\Sigma M_O(\bar{F}_k) = 0$

$$-\Phi_1 \cdot \frac{2}{3}a\cos\varphi + m_1 g \frac{a}{2}\sin\varphi - m_2 g \frac{b}{2}\cos\varphi + \Phi_2 \cdot \frac{2}{3} b\sin\varphi = 0$$

$$-\cancel{m_1}\omega^2 \frac{a}{2}\sin\varphi \cdot \frac{2}{3}a\cos\varphi + \cancel{m_1} g \frac{a}{2}\sin\varphi - \cancel{m_1}\frac{b}{a} g \cdot \frac{b}{2}\cos\varphi + \cancel{m_1}\frac{b}{a}\omega^2 \frac{b}{2}\cos\varphi \cdot \frac{2}{3} b\sin\varphi = 0$$

$$\omega^2 = \frac{\frac{1}{2}\left(\frac{b^2}{a}\cos\varphi - a\sin\varphi\right) g}{\frac{1}{6}\sin 2\varphi\left(\frac{b^3}{a} - a^2\right)} = \frac{3g(b^2\cos\varphi - a^2\sin\varphi)}{\sin\varphi(b^3 - a^3)}$$

Д III 007



стержень $OA = l$, m_1, точка A — m_2, угол φ

ω — ?

Силы инерции: $\Phi_1 = m_1 \cdot a_c^{ц} = m_1 \omega^2 \frac{l}{2}\sin\varphi$

$\Phi_2 = m_2 a_A^{ц} = m_2 \omega^2 l \sin\varphi$

по принципу Даламбера $(m_1\bar{g}, m_2\bar{g}, \bar{R}_O, \bar{\Phi}_1, \bar{\Phi}_2) \backsim 0$

$\Sigma M_O(\bar{F}_k) = 0$

$$\Phi_2 l\cos\varphi + \Phi_1 \cdot \frac{2}{3} l\cos\varphi - m_2 g l\sin\varphi - m_1 g \frac{l}{2}\sin\varphi = 0$$

$$m_2\omega^2 l\cancel{\sin\varphi}\cdot\cos\varphi + m_1\omega^2\frac{l}{2}\cancel{\sin\varphi}\cdot\frac{2}{3}\cos\varphi - \left(\frac{m_1}{2} + m_2\right) g\cdot\cancel{\sin\varphi} = 0$$

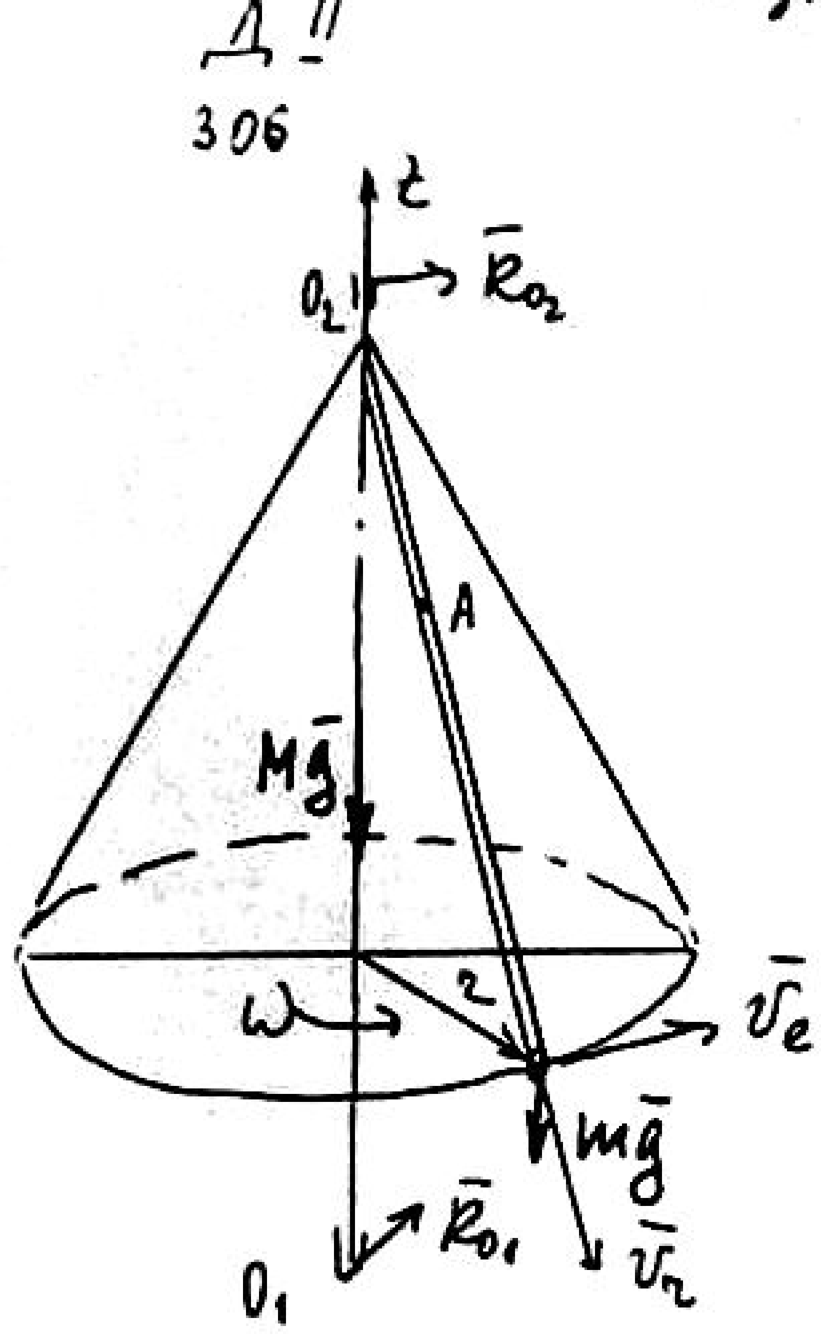
$$\omega^2 = \frac{\left(\frac{m_1}{2} + m_2\right) g}{l\cos\varphi\left(\frac{m_1}{3} + m_2\right)}$$

+

Теорема об изменении кинетического момента $\frac{dK_z}{dt} = \sum M_z(\bar{F}_k^{(e)})$

(закон сохранения кинетического момента, когда $\sum M_z(\bar{F}_k^{(e)}) = 0$)

Д II
306



m, $J_z = 3mr^2$ – конус, шарик на вершине конуса $\omega = \omega_0$

шарик внизу ω – ?

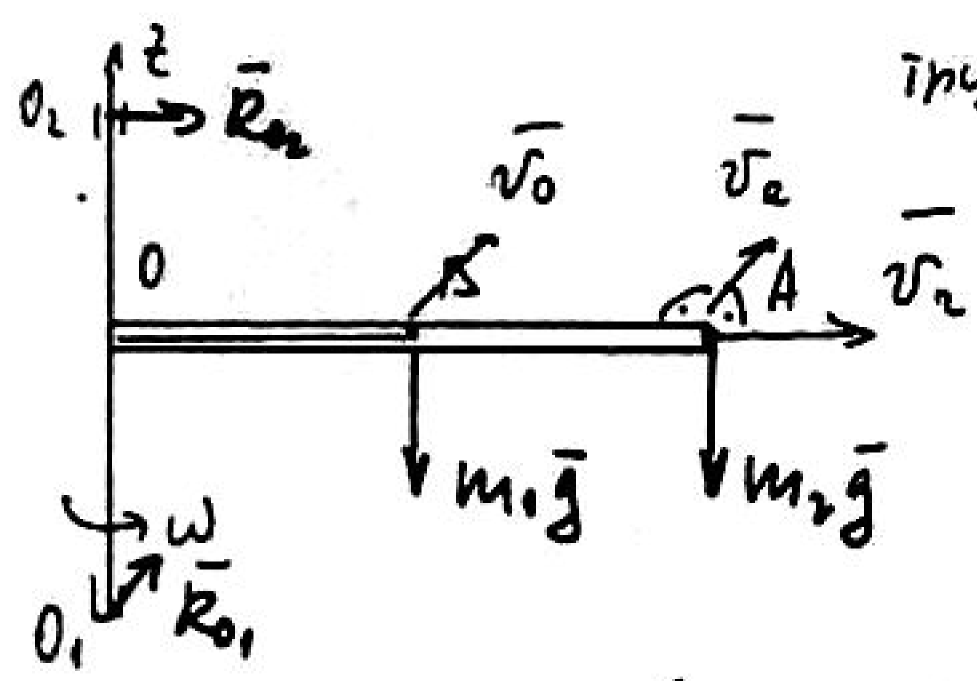
$$\frac{dK_z}{dt} = \sum M_z(\bar{F}_k^{(e)}) = 0 \qquad K_z = const$$

$$K_{z0} = J_z \omega_0 = 3mr^2\omega_0$$

$$K_z = J_z\omega + m v_e \cdot r, \quad v_e = \omega r$$

$$K_z = J_z\omega + mr^2\omega = 4mr^2\omega$$

$$3mr^2\omega_0 = 4mr^2\omega, \quad \boxed{\omega = \frac{3}{4}\omega_0}$$



трубка $OA = 2l$, масса m_1, шарик m_2

в начальном положении $OB = l$, $\omega = \omega_0$

в конечном положении $OA = 2l$, ω – ?

$$\frac{dK_z}{dt} = \sum M_z(\bar{F}_k^{(e)}) = 0 \qquad K_z = const$$

в начальном положении $\omega = \omega_0$, $v_0 = \omega_0 l$

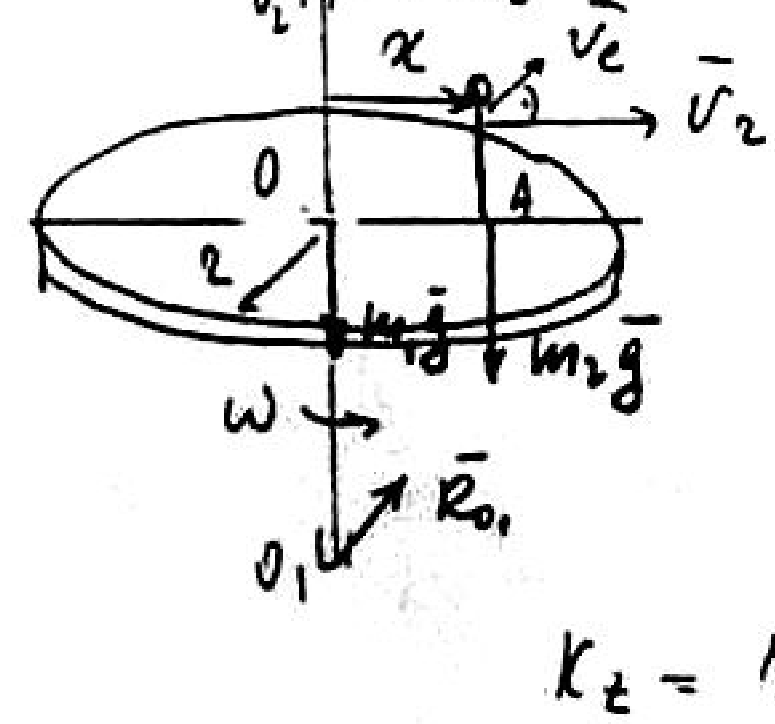
$$K_{z0} = J_z\omega_0 + m v_0 l = \frac{1}{3}m_1(2l)^2\omega_0 + m_2\omega_0 l^2 = \left(\frac{4}{3}m_1 + m_2\right)l^2\omega_0$$

в конечном положении ω, $v_e = \omega \cdot 2l$

$$K_z = J_z\omega + m \cdot v_e \cdot 2l = \frac{4}{3}m_1 l^2\omega + m_2\omega \cdot 4l^2 = \left(\frac{4}{3}m_1 + 4m_2\right)l^2\omega$$

$$\left(\frac{4}{3}m_1 + m_2\right)l^2\omega_0 = \left(\frac{4}{3}m_1 + 4m_2\right)l^2\omega; \quad \boxed{\omega = \frac{\frac{4}{3}m_1 + m_2}{\frac{4}{3}m_1 + 4m_2} \cdot \omega_0}$$

Д II
304



платформа m_1, r, ω_0 – когда человек на оси z

человек m_2, пошёл к краю

$\omega = \omega(x)$ – ?

$$\frac{dK_z}{dt} = \sum M_z(\bar{F}_k^{(e)}) = 0 \qquad K_z = const$$

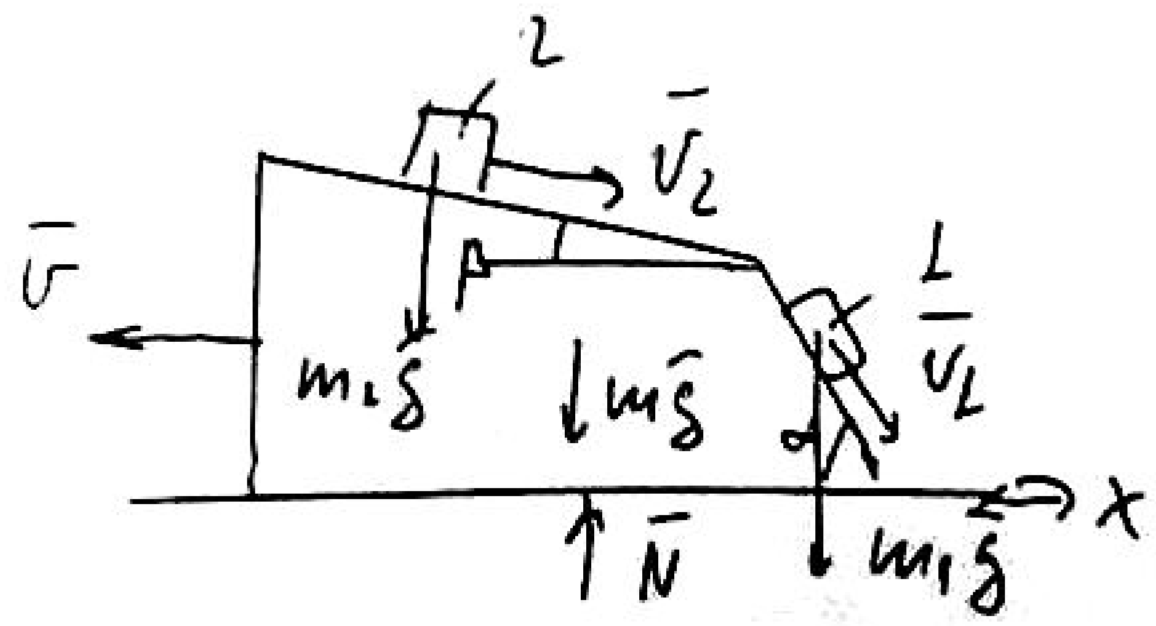
$$K_{z0} = J_z\omega_0 = \frac{m_1 r^2}{2}\omega_0$$

$$K_z = J_z\omega + m v_e \cdot x, \qquad v_e = \omega x$$

$$K_z = \frac{m_1 r^2}{2}\omega + m x^2\omega, \qquad K_{z0} = K_z$$

$$\boxed{\omega = \frac{m_1 r^2}{m_1 r^2 + 2m_2 x^2} \cdot \omega_0}$$

Теорема об изменении количества движения $\frac{dQ}{dt}=\sum F_k^{(e)}$



m, m_1, m_2, покой

v_1, v_2 – относит. скорости грузов.

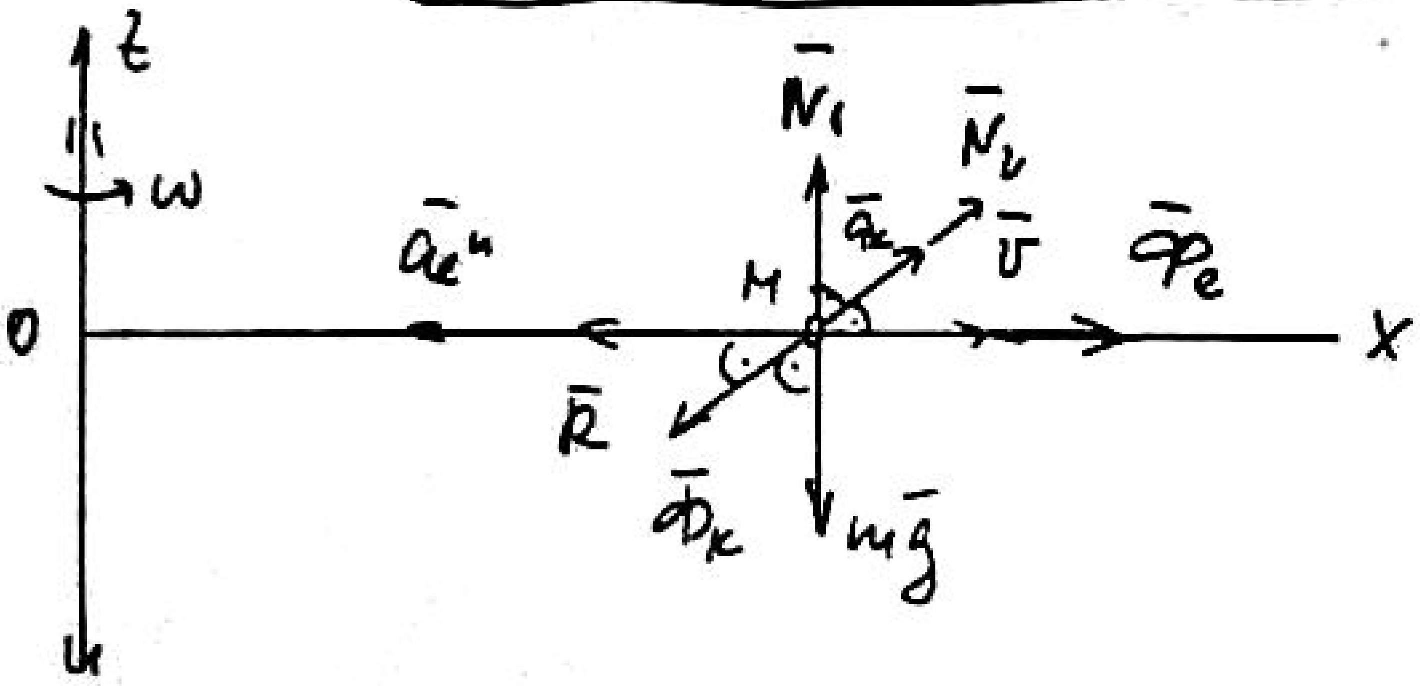
v – скорость призмы!

$\frac{dQ_x}{dt} = \sum F_{x}^{(e)} = 0$ на ось x

$Q_x = const = Q_{x0} = 0$, $Q_x = -(m+m_1+m_2)v + m_1 v_1\cos\alpha + m_2 v_2\cos\beta = 0$

$$\boxed{v = \frac{m_1 v_1 \cos\alpha + m_2 v_2 \cos\beta}{m_1 + m_2 + m}}$$

Динамика сложного движения точки | НИСО



$\omega = const$, $R = \mu v$ сила сопротивления, v – относит. скорость

$t=0$, $v=0$, $x=l$.

Определить $x = x(t)$

Силы инерции: $\bar\Phi_e = -m\bar a_e^n$; $a_e^n = \omega^2 x$

$\Phi_e = m\omega^2 x$; $\bar\Phi_k = -m\bar a_k$; $a_k = 2\omega v_r = 2\omega\dot x$

$\Phi_k = 2m\omega\dot x$

$m\bar a_r = m\bar g + \bar N_1 + \bar N_2 + \bar R + \bar\Phi_e + \bar\Phi_k$

на x: $m\ddot x = \sum F_{kx} = \Phi_e - R = m\omega^2 x - \mu\dot x$; $\ddot x + \frac{\mu}{m}\dot x - \omega^2 x = 0$

Характеристическое уравнение: $\lambda^2 + \frac{\mu}{m}\lambda - \omega^2 = 0$; $\lambda_{1,2} = -\frac{\mu}{2m} \pm \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}$,

$x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$; $\dot x = \lambda_1 C_1 e^{\lambda_1 t} + \lambda_2 C_2 e^{\lambda_2 t}$

$\lambda_1 = -\frac{\mu}{2m} + \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}$,

$\lambda_2 = -\frac{\mu}{2m} - \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}$

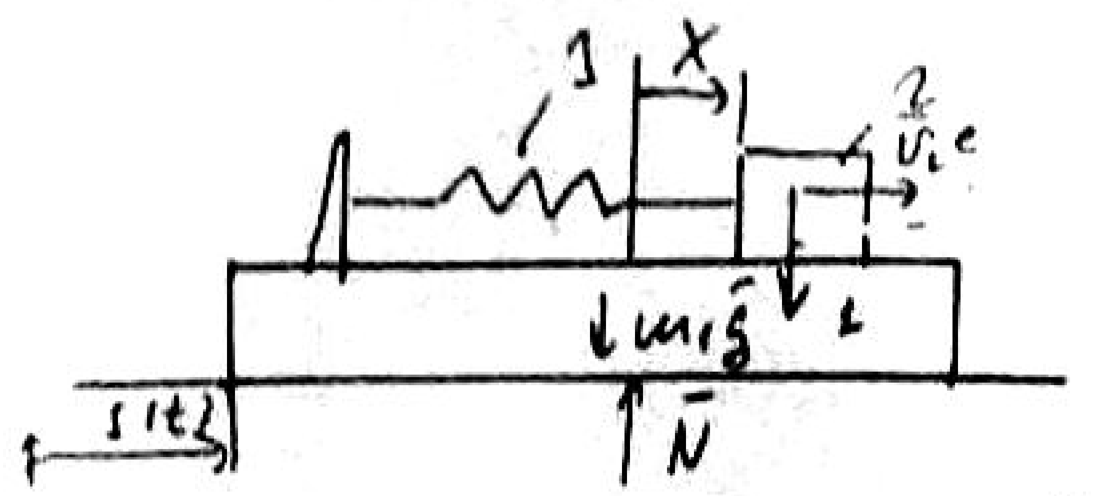
н.у. $t=0$, $x=l$, $\dot x = 0$; $l = C_1 + C_2$, $C_2 = l - C_1$

$0 = \lambda_1 C_1 + \lambda_2 C_2$, $\lambda_1 C_1 + \lambda_2(l - C_1) = 0$ $\lambda_1 C_1 + \lambda_2 l - \lambda_2 C_1 = 0$

$$C_1 = \frac{\lambda_2 l}{\lambda_2 - \lambda_1} = \frac{-\frac{\mu}{2m} - \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}{-\cancel{\frac{\mu}{2m}} - \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2} + \cancel{\frac{\mu}{2m}} - \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}\, l = \frac{\frac{\mu}{2m} + \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}{2\sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}} \cdot l$$

$$C_2 = l - C_1 = l\left(1 - \frac{\frac{\mu}{2m} + \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}{2\sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}\right) = \frac{-\frac{\mu}{2m} + \sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}{2\sqrt{\left(\frac{\mu}{2m}\right)^2 + \omega^2}}\, l$$

Уравнение Лагранжа II рода



m_1, m_2, c

$s(t) = s_0 \sin pt$

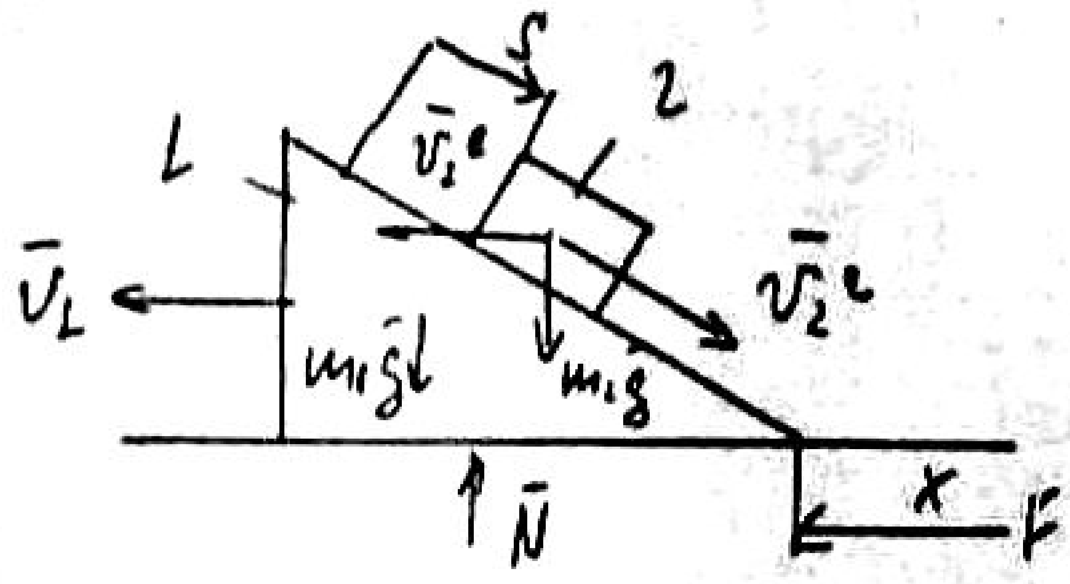
$F_{пр} = cx$

$v_1 = \dot s = v_2^e, \quad v_2^r = \dot x$

$T_1 = \frac12 m_1 v_1^2 = \frac12 m_1 \dot s^2, \quad T_2 = \frac12 m_2 v_2^2 = \frac12 m_2 (\dot x + \dot s)^2$

$T = \frac12 (m_1+m_2)\dot s^2 + m_2 \dot x \dot s + \frac12 m_2 \dot x^2, \qquad \dot s = s_0 p \cos pt$

$Q_s = \frac{\sum[\delta A(\bar F_k)]_s}{\delta s} = 0, \qquad Q_x = \frac{\sum[\delta A(\bar F_k)]_x}{\delta x} = -\frac{F_{пр}\,\delta x}{\delta x} = -cx$



m_1, m_2, α

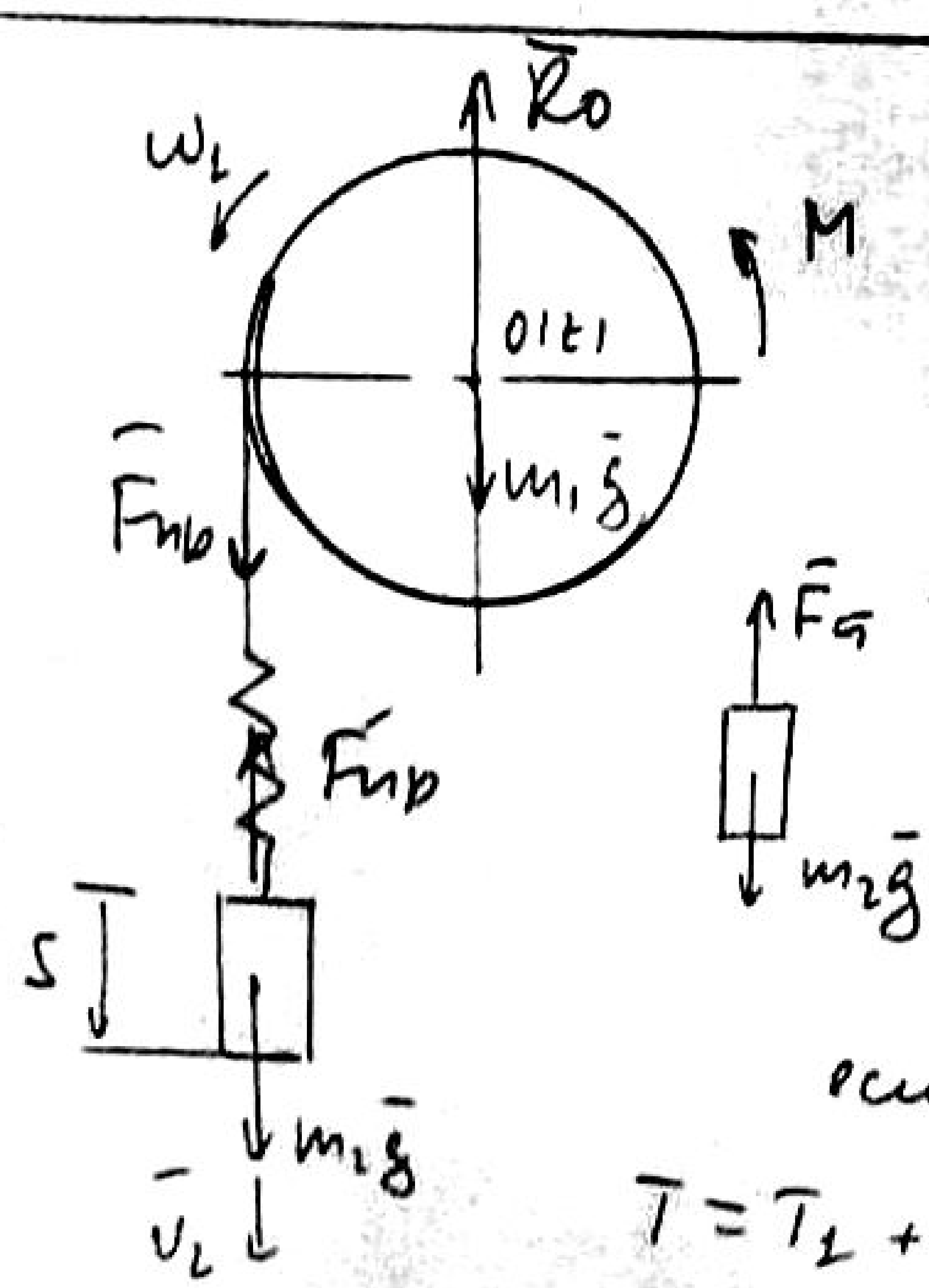
$v_1 = \dot x = v_2^e, \quad v_2^r = \dot s$

$T_1 = \frac12 m_1 v_1^2 = \frac12 m_1 \dot x^2$

$T_2 = \frac12 m_2 v_2^2 = \frac12 m_2 (\dot x^2 + \dot s^2 - 2\dot x \dot s \cos\alpha)$

$T = \frac12 (m_1+m_2)\dot x^2 - 2m_2 \dot x \dot s \cos\alpha + \frac12 m_2 \dot s^2$

$Q_x = \frac{\sum[\delta A(\bar F_k)]_x}{\delta x} = 0, \qquad Q_s = \frac{\sum[\delta A(\bar F_k)]_s}{\delta s} = \frac{m_2 g \sin\alpha\, \delta s}{\delta s} = m_2 g \sin\alpha$



Дано: $m_1, r, m_2, M = M_0 \sin pt, c$

нач. координаты s — от положения статического равновесия.

положение стат. равновесия

$\sum F_{ks} = 0 \quad m_2 g - F_{ст} = 0, \quad m_1 g = F_{ст} = c\Delta_{ст}$ (1)

$\Delta_{ст} = \frac{m_1 g}{c}$

$\omega_1 = \dot\varphi, \quad v_1 = \dot s$

сила пружины $F_{пр} = c\Delta, \quad \Delta = \Delta_{ст} + s - r\varphi$

если $\Delta > 0$, то пружина растянута

$T = T_1 + T_2, \quad T_1 = \frac12 J_{oz}\omega_1^2 = \frac12 \frac{m_1 r^2}{2}\dot\varphi^2, \quad T_2 = \frac12 m_2 v_2^2 = \frac12 m_2 \dot s^2$

$T = \frac14 m_1 r^2 \dot\varphi^2 + \frac12 m_2 \dot s^2$

$Q_\varphi = \frac{\sum[\delta A(\bar F_k)]_\varphi}{\delta\varphi}, \quad \delta s = 0, \ \delta\varphi \ne 0, \qquad Q_\varphi = \frac{M\delta\varphi + F_{пр} \cdot r\,\delta\varphi}{\delta\varphi} = M + F_{пр} r =$

$= M + c(\Delta_{ст} + s - r\varphi)$

$Q_s = \frac{\sum[\delta A(\bar F_k)]_s}{\delta s}, \quad \delta\varphi = 0, \ \delta s \ne 0, \qquad Q_s = \frac{m_1 g\,\delta s - F_{пр}\,\delta s}{\delta s} = m_1 g - F_{пр} =$

$= \cancel{m_1 g} - c(\cancel{\Delta_{ст}} + s - r\varphi) = -c(s - r\varphi)$

$F_k \cdot \bar{r}_k$

$T = \frac{1}{2} J_c \omega^2 + \frac{1}{2} m_2 v_c^2 + \frac{1}{2} J_2 \omega_2^2$

$\omega_2 = \frac{v_A}{r_2}$ $\quad v_D = 2\omega_2 r_2$ $\quad \omega_1 \cdot z_1 = 2\omega_2 r_2$

$\omega_1 = \frac{2\omega_2 r_2}{r_1} = \frac{2 v_A}{z_1}$

$T = \frac{1}{4} m_1 z_1^2 \frac{4 v_A^2}{z_1^2} + \frac{1}{2} m_2 v_A^2 + \frac{1}{4} m_2 z_2^2 \frac{v_A^2}{r_2^2} =$

$= \left(m_1 + \frac{1}{2} m_2 + \frac{1}{4} m_2\right) v_A^2 = \frac{1}{2}\left(2m_1 + \frac{3}{2} m_2\right) v_A^2$

$dT = \left(2m_1 + \frac{3}{2} m_2\right) v_A dv_A = \left(2m_1 + \frac{3}{2} m_2\right) a_A dS$

$d'A = M d\varphi = M \frac{dS r_2}{r_1}.$

$\omega r = v_A$ $\quad 2d\varphi\, r = dS$

$2\omega r = v_D$

$dS = d\varphi \cdot r_2.$ $\quad \frac{d\varphi r_2}{r_1}$

$dS = d\varphi \cdot r$

$d\varphi_1 = \frac{dS r_2}{r_1}$

$Q = -F\delta x_1 + P\delta x_2 = -F\delta x_1 +$

$F \frac{\partial r}{\partial q_i}$

$Q = \sum_{k=1}^{n} \bar{F}_k \frac{\partial \bar{r}_k}{\partial q_i}$

$x_1 = \frac{x}{2}$ $\quad x_2$

$Q = +F\frac{1}{2} - P + Q\frac{1}{2} = 0$

$P = \frac{1}{2}(F + Q).$

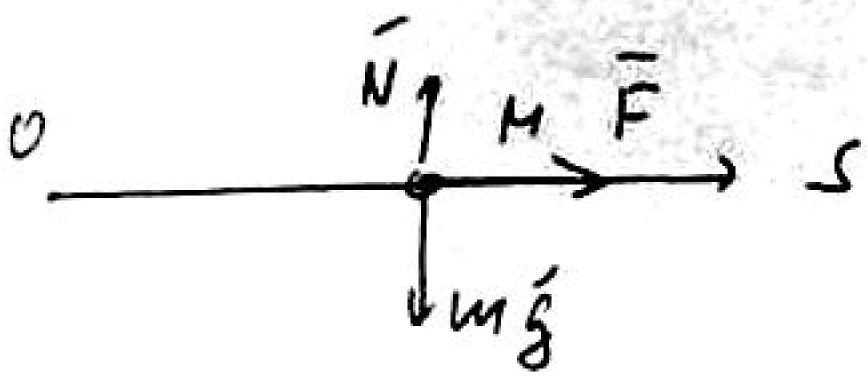
$A = -\Pi = -\frac{c \cdot ?}{2}$

А

Динамика точки.

Пример m, $v_0=0$, $F=F_0(1-\frac{s}{l})$, s – текущая координата

v при $s=l$ – ?



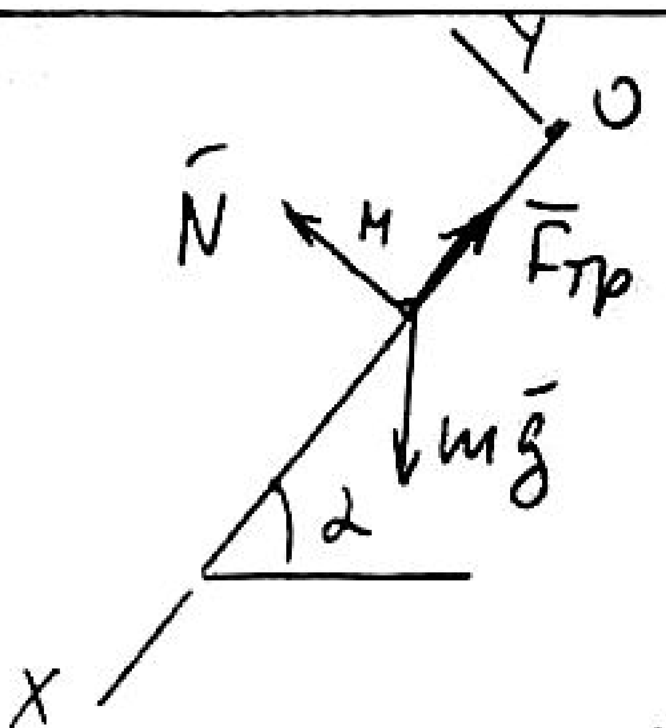
$m\bar a = m\bar g + \bar N + \bar F$, в проекции на s

$m\frac{dv}{dt} = F = F_0(1-\frac{s}{l})$; $\frac{dv}{dt} = \frac{F_0}{m}(1-\frac{s}{l})$

$\frac{dv}{dt}\cdot\frac{ds}{ds} = \frac{ds}{dt}\cdot\frac{dv}{ds} = v\frac{dv}{ds} = \frac{F_0}{m}(1-\frac{s}{l})$; $v\,dv = \frac{F_0}{m}(1-\frac{s}{l})\,ds$, интегрируем

$\frac{v^2}{2} = \frac{F_0}{m}(s-\frac{s^2}{2l}) + C_1$; н.у. $s=0$ $v=0$ $C_1=0$

$v^2 = \frac{2F_0}{m}(s-\frac{s^2}{2l})$; при $s=l$ $v=\sqrt{\frac{F_0}{m}l}$



m, $\alpha=45^\circ$, $f=0{,}5$ $v_0=0$

$x=x(t)$ – ?

$m\bar a = m\bar g + \bar N + \bar F_{тр}$; $ma_x = mg\sin\alpha - F_{тр}$,

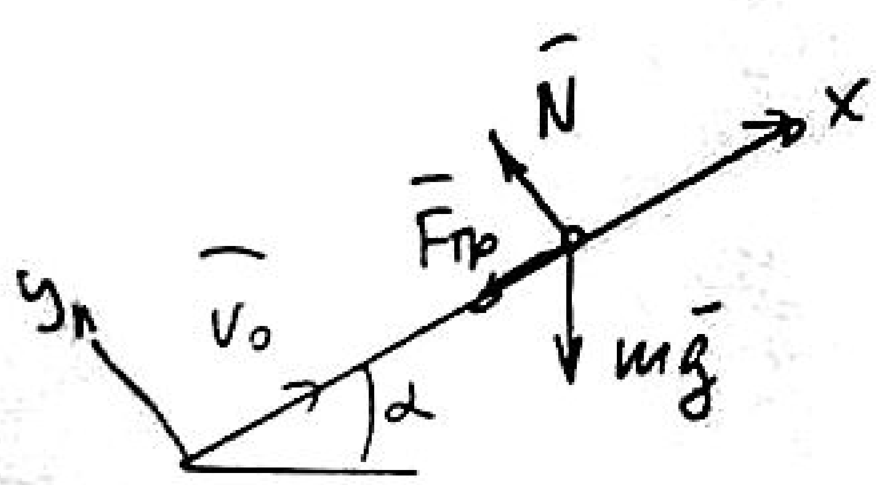
$ma_y = N - mg\cos\alpha = 0$, $N = mg\cos\alpha$, $F_{тр} = fN = fmg\cos\alpha$

$ma_x = m\frac{dv_x}{dt} = mg\sin\alpha - fmg\cos\alpha = mg(\sin\alpha - f\cos\alpha) = mg(\frac{\sqrt2}{2} - \frac{\sqrt2}{4}) = mg\frac{\sqrt2}{4}$

$\frac{dv_x}{dt} = mg\frac{\sqrt2}{4}$; $v_x = mg\frac{\sqrt2}{4}t + C_1$, н.у. $t=0$ $v_x=0$ $C_1=0$

$v_x = \frac{dx}{dt} = mg\frac{\sqrt2}{4}t$, $x = mg\frac{\sqrt2}{8}t^2 + C_2$, н.у. $t=0$ $x=0$ $C_2=0$

$x = mg\frac{\sqrt2}{8}t^2$ (м)



m, $\alpha=30^\circ$, $f=\frac{\sqrt3}{3}$, v_0, $v=0$ путь?

$m\frac{d\bar v}{dt} = m\bar g + \bar N + \bar F_{тр}$, $m\frac{dv_x}{dt} = -mg\sin\alpha - F_{тр}$

$m\frac{dv_y}{dt} = N - mg\cos\alpha = 0$, $N = mg\cos\alpha$. $F_{тр} = fN = fmg\cos\alpha$

$\cancel{m}\frac{dv_x}{dt} = -mg\sin\alpha - fmg\cos\alpha = -mg(\sin\alpha + f\cos\alpha) = -mg(\frac{1}{2} + \frac{\sqrt3}{3}\cdot\frac{\sqrt3}{2}) = -\cancel{m}g$

$\frac{dv_x}{dt}\cdot\frac{dx}{dx} = v_x\frac{dv_x}{dx} = -g$; $v_x\,dv_x = -g\,dx$, интегрируем

$\frac{v_x^2}{2} = -gx + C_1$, н.у. $x=0$ $v_x=v_0$, $\frac{v_0^2}{2} = C_1$

$v_x^2 = v_0^2 - 2gx$, при $v_x=0$ $x=s$

$$\boxed{s = \frac{v_0^2}{2g}}$$

12) + Теорема об изменении кинет. энергии (11)

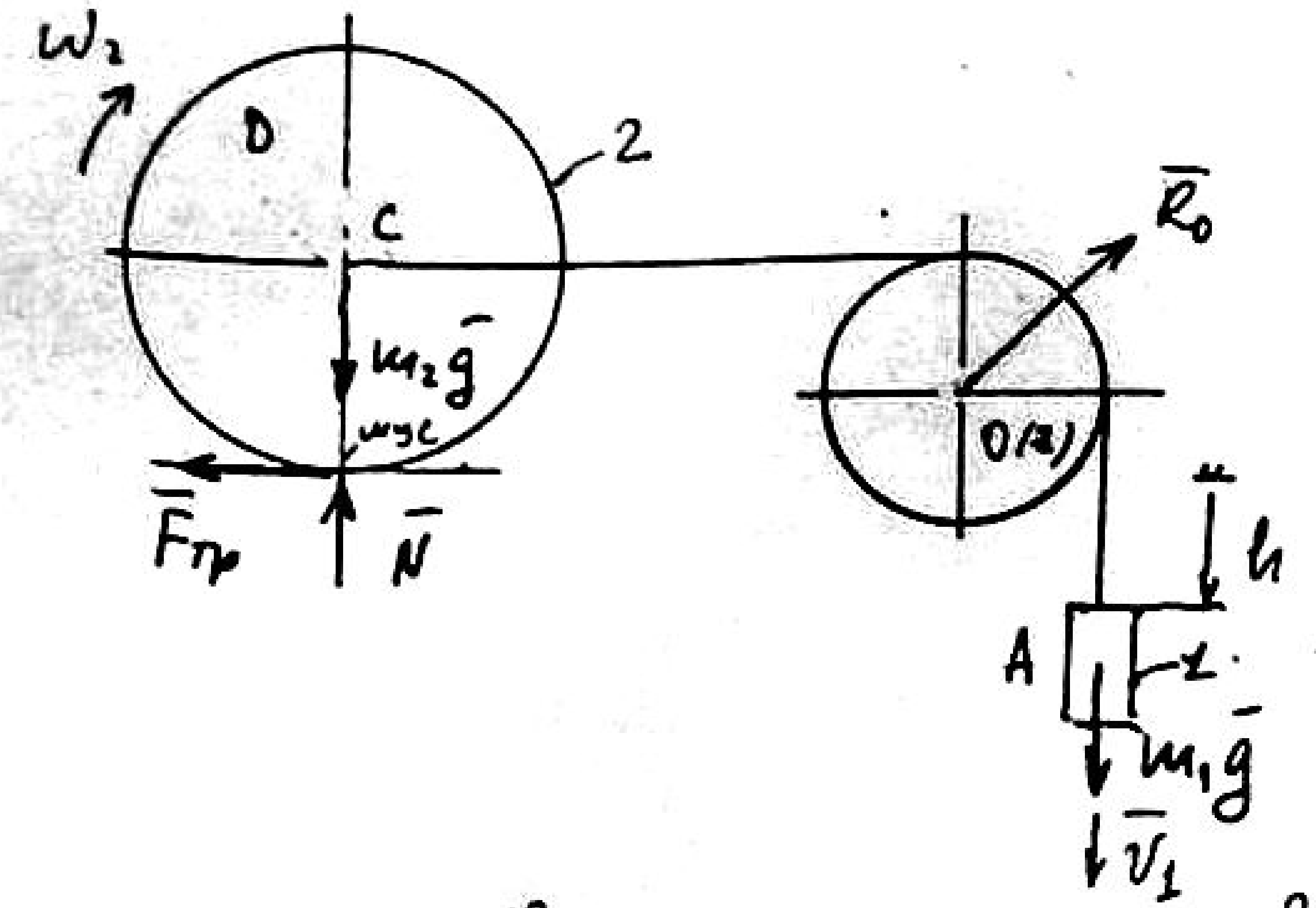
Д II
407

груз A – m_1, блок невесом

каток D – m_2, R, катится без скольжения

покой $T_0=0$. Груз опустится на высоту h

V_1 – ?



груз 1 – поступательное движение

V_1 – скорость груза

каток 2 – плоское движение

$V_C = V_1$ скорость центра масс катка

$\omega_2 = \frac{V_C}{R} = \frac{V_1}{R}$ угловая скорость катка

$$T - \cancel{T_0}^{\,0} = \sum A(\bar{F}_k^{(e)}) + \cancel{\sum A(\bar{F}_k^{(i)})}^{\,0}; \quad \text{кинетическая энергия } T = T_1 + T_2$$

$$T_1 = \frac{1}{2} m_1 V_1^2; \quad T_2 = \frac{1}{2} m_2 V_C^2 + \frac{1}{2} J_{C2}\omega_2^2 = \frac{1}{2} m_2 V_1^2 + \frac{1}{2}\frac{m_2 R^2}{2}\cdot\left(\frac{V_1}{R}\right)^2 = \frac{3}{4} m_2 V_1^2$$

$T = \left(\frac{m_1}{2} + \frac{3}{4} m_2\right) V_1^2$; работа внешних сил $\sum A(\bar{F}_k^{(e)}) = m_1 g h$

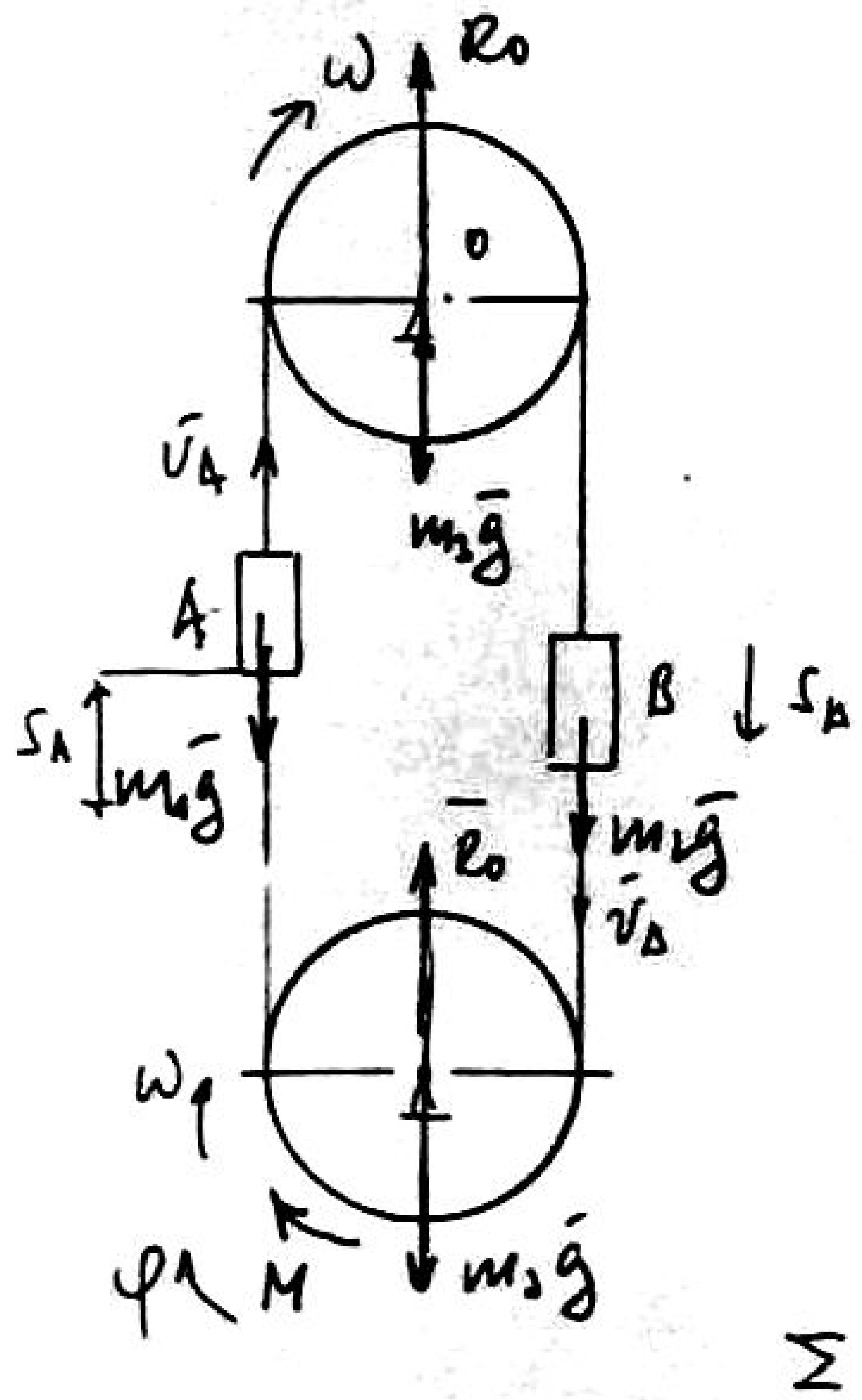
$$\left(\frac{m_1}{2} + \frac{3}{4} m_2\right) V_1^2 = m_1 g h \qquad V_1 = \sqrt{\frac{m_1 g h}{\frac{m_1}{2} + \frac{3}{4} m_2}}$$

Д II
420

два шкива, массой m_3, радиус r, радиус инерции ρ

груз A – m_1, груз B – m_2, $M = \text{const}$ момент

$V_A = V_A(S_A)$ – ?



$$T - \cancel{T_0}_{\,0} = \sum A(\bar{F}_k^{(e)}) + \cancel{\sum A(\bar{F}_k^{(i)})}^{\,0}$$

V_A – скорость груза A, $V_B = V_A$

$\omega = \frac{V_A}{r}$ угловая скорость шкивов

$$T = T_1 + T_2 + 2T_3; \quad T_1 = \frac{1}{2} m_1 V_A^2, \quad T_2 = \frac{1}{2} m_2 V_B^2 = \frac{1}{2} m_2 V_A^2$$

$$T_3 = \frac{1}{2} J_O \omega^2 = \frac{1}{2} m_3 \rho^2 \left(\frac{V_A}{r}\right)^2$$

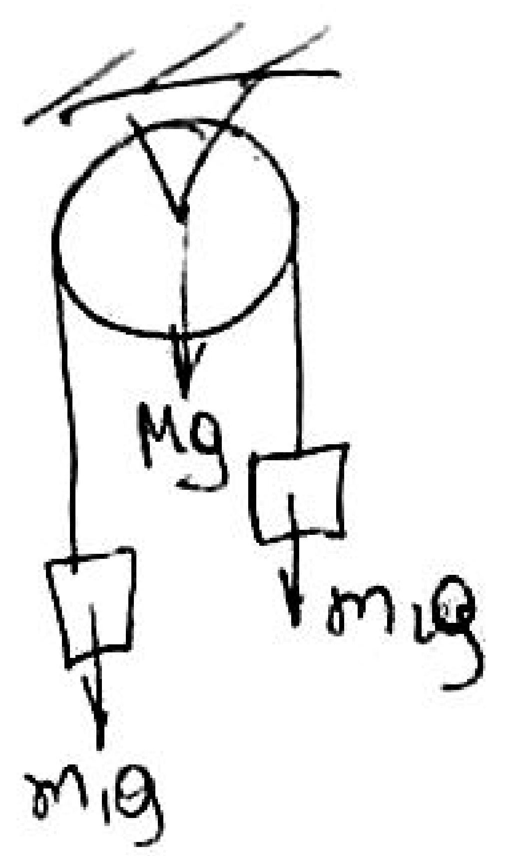
$$T = \left[\frac{m_1 + m_2}{2} + m_3\left(\frac{\rho}{r}\right)^2\right] V_A^2$$

$$\sum A(\bar{F}_k^{(e)}) = M\varphi - m_1 g S_A + m_2 g S_B; \qquad S_B = S_A,\ \varphi = \frac{S_A}{r}$$

$$\sum_k A(\bar{F}_k^{(e)}) = M\frac{S_A}{r} + (m_2 - m_1) g S_A$$

$$\left[\frac{m_1 + m_2}{2} + m_3\left(\frac{\rho}{r}\right)^2\right] V_A^2 = \left[\frac{M}{r} + (m_2 - m_1) g\right] S_A$$

$$V_A = \sqrt{\frac{M/r + (m_2 - m_1) g}{\frac{m_1 + m_2}{2} + m_3\left(\frac{\rho}{r}\right)^2}\, S_A};$$



$$T = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{1}{2} J_{Oz}\omega^2$$

$$v_1 = v_2$$

$$T = \left(\frac{m_1}{2} + \frac{m_2}{2}\right) v_1^2 + \frac{1}{2}\cdot\frac{1}{2} MR^2\omega^2 =$$

$$v_1 = \omega R = \ldots$$

$$T = \left(\frac{m_1}{2} + \frac{m_2}{2}\right)\omega^2 R^2 + \frac{1}{4} MR^2\omega^2 =$$

$$= \left(\left(\frac{m_1}{2} + \frac{m_2}{2}\right) R^2 + \frac{1}{4} MR^2\right)\omega^2$$

$$dT = \cancel{R^2(m} \ \frac{1}{\cancel{2}}\left(\left(m_1 + m_2 + \frac{M}{2}\right) R^2\right)\omega\, d\omega$$

$$T = \left(\frac{m_1}{2} + \frac{m_2}{2}\right)\omega^2 r^2 + \frac{1}{2} M\rho^2\omega^2 = \frac{1}{2}\left((m_1+m_2)r^2 + M\rho^2\right)\omega^2$$

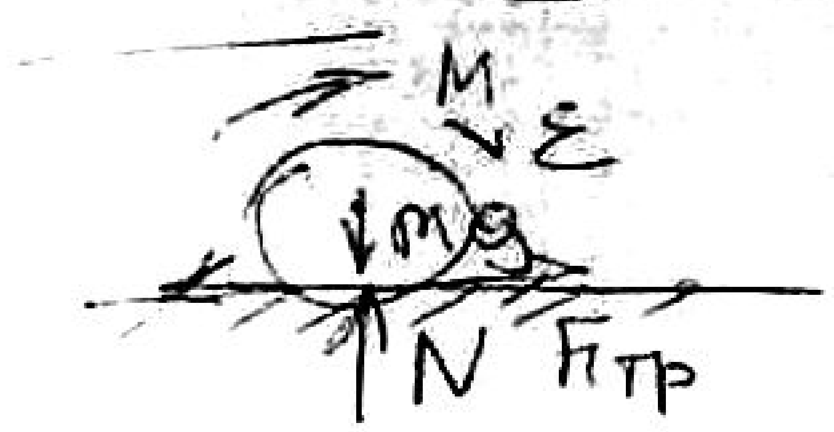
$$dT = \left((m_1+m_2)r^2 + M\rho^2\right)\omega\, d\omega$$

$$d'A = +m_1 g\, ds - m_2 g\, ds = (m_1 - m_2) g\, r\, d\varphi$$

$$\omega\, d\omega = \omega \frac{d\omega}{dt} dt = d\varphi\, \varepsilon$$

$$\left((m_1+m_2)r^2 + M\rho^2\right)\varepsilon = (m_1 - m_2) g r$$

$$\varepsilon = (m_1 - m_2)\cdot$$



$$J_{Oz}\varepsilon = M - F_{тр}\cdot r$$

$$\frac{1}{2} mR^2\varepsilon = M - F_{тр}\cdot r$$

$$\varepsilon = \frac{2M - F_{тр} r}{mR^2}$$

$$ma = +F_{тр}$$

$$ma = f mg$$

$$\varepsilon r = a$$

$$m\varepsilon r = f mg$$

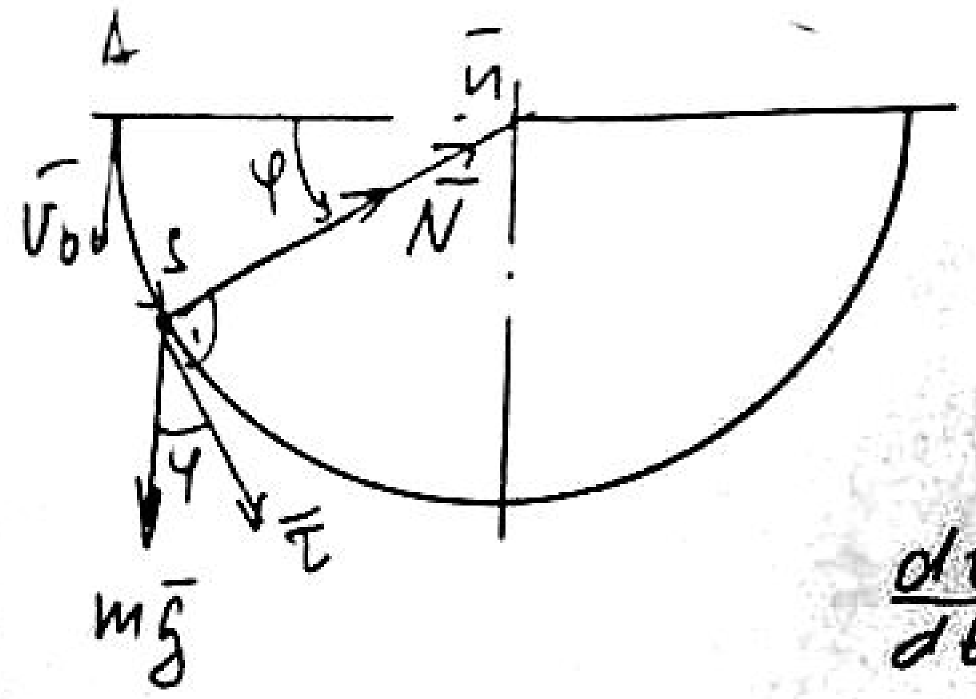
$$J\varepsilon = -\cancel{F_{тр}}\ \delta mg$$

$$\varepsilon = \frac{\delta mg}{J}$$

+

Динамика точки

2



m, $v_0=\sqrt{gR}$, R,

v при $\varphi=\frac{2}{3}\pi$!

$m\bar a = m\bar g + \bar N$, на $\bar\tau$: $m a_\tau = m\frac{dv}{dt} = F_\tau = mg\cos\varphi$

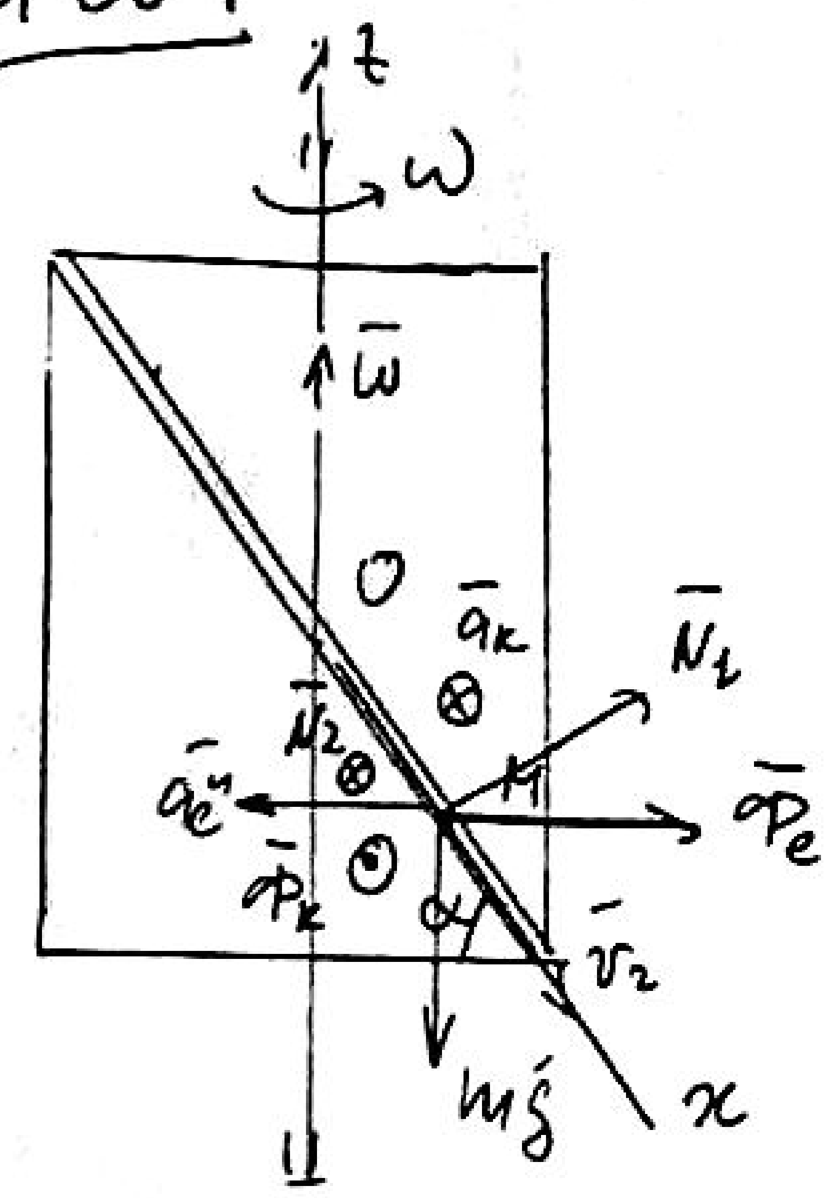
$\frac{dv}{dt} = g\cos\varphi$; $\frac{dv}{dt}\cdot\frac{ds}{Rd\varphi} = \frac{ds}{dt}\cdot\frac{dv}{Rd\varphi} = v\frac{dv}{Rd\varphi} = g\cos\varphi$

$v\,dv = gR\cos\varphi\,d\varphi$, интегрируя $\frac{v^2}{2} = gR\sin\varphi + C_1$, н.у. $\varphi=0$ $v=v_0$

$\frac{v_0^2}{2} = C_1$; $v^2 = v_0^2 + 2gR\sin\varphi$, при $\varphi=\frac{2}{3}\pi$ $\sin\varphi=\frac{\sqrt3}{2}$

$v^2 = gR + gR\sqrt3$; $\boxed{v=\sqrt{gR(1+\sqrt3)}}$

НИСО



Дано: m, α, $t=0$ $x_0=l$, $\dot x_0=0$ $\omega=\text{const}$

$x = x(t)$ - ?

$\bar\Phi_e = -m\bar a_e$, $a_e = a_e^n = \omega^2 x\cos\alpha$

$\Phi_e = m\omega^2 x\cos\alpha$

$\bar\Phi_k = -m\bar a_k$, $\bar a_k = 2(\bar\omega\times\bar v_2)$,

$a_k = 2\omega v_2\sin(90^\circ+\alpha) = 2\omega v_2\cos\alpha = 2\omega\dot x\cos\alpha$

$\Phi_k = 2m\omega\dot x\cos\alpha$

$m\bar a_2 = m\bar g + \bar N_1 + \bar N_2 + \bar\Phi_e + \bar\Phi_k$, в проекции на ось x

$m\ddot x = \Sigma F_{kx} = mg\sin\alpha + \Phi_e\cos\alpha = mg\sin\alpha + m\omega^2 x\cos^2\alpha$; $\omega\cos\alpha = \omega_1$

$\ddot x = g\sin\alpha + \omega_1^2 x$; $\boxed{\ddot x - \omega_1^2 x = g\sin\alpha}$ $x = x_{oo} + x_{чн}$;

$\lambda^2 - \omega_1^2 = 0$ $\lambda_{1,2} = \pm\omega_1$, $x_{oo} = C_1 e^{\omega_1 t} + C_2 e^{-\omega_1 t}$

$x_{чн} = B = \text{const}$, $0 - \omega_1^2 B = g\sin\alpha$, $B = -\frac{g\sin\alpha}{\omega_1^2}$

$x = C_1 e^{\omega_1 t} + C_2 e^{-\omega_1 t} - \frac{g\sin\alpha}{\omega_1^2}$ | н.у. $t=0$ $x_0=l$, $\dot x_0=0$

$\dot x = \omega_1(C_1 e^{\omega_1 t} - C_2 e^{-\omega_1 t})$

$l = C_1 + C_2 - \frac{g\sin\alpha}{\omega_1^2}$; $0=\omega_1(C_1-C_2)$, $C_1=C_2=\frac{l}{2}+\frac{g\sin\alpha}{2\omega_1^2}$

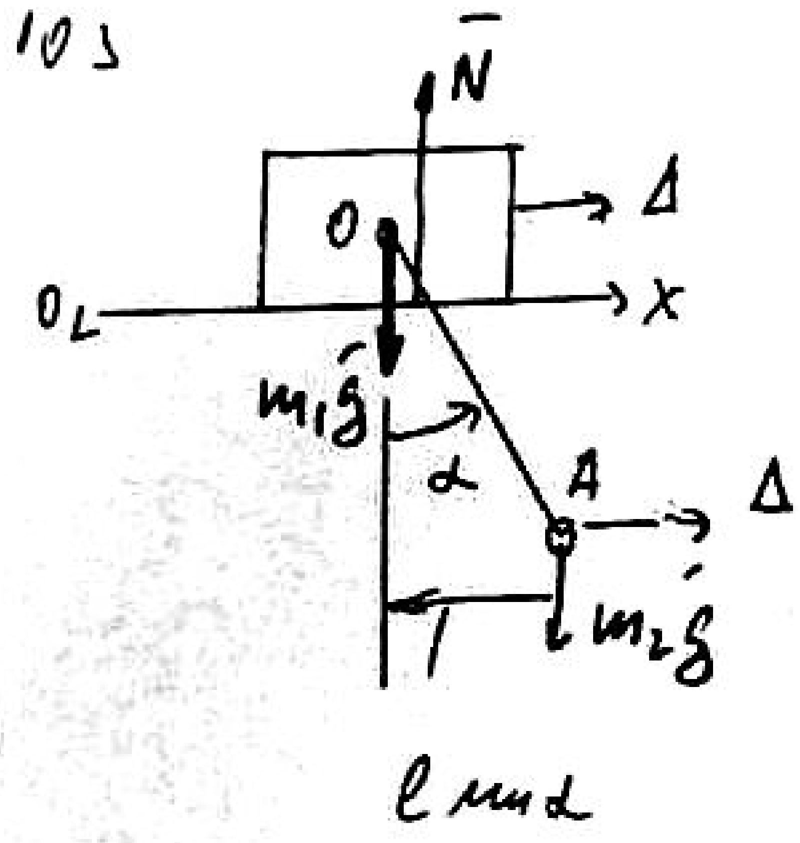
$x = \left(l+\frac{g\sin\alpha}{\omega_1^2}\right)\cdot\frac{e^{\omega_1 t}+e^{-\omega_1 t}}{2} - \frac{g\sin\alpha}{\omega_1^2} = \left(l+\frac{g\sin\alpha}{\omega_1^2}\right)\operatorname{ch}(\omega_1 t) - \frac{g\sin\alpha}{\omega_1^2}$

Теорема о движении центра масс $M\bar{a}_c = \sum \bar{F}_k^{(e)}$

Д II 103

Дано: m_1, $OA = l$, m_2, $\alpha = 30°$, покой

Δ при $\alpha = 0$?



Решение.

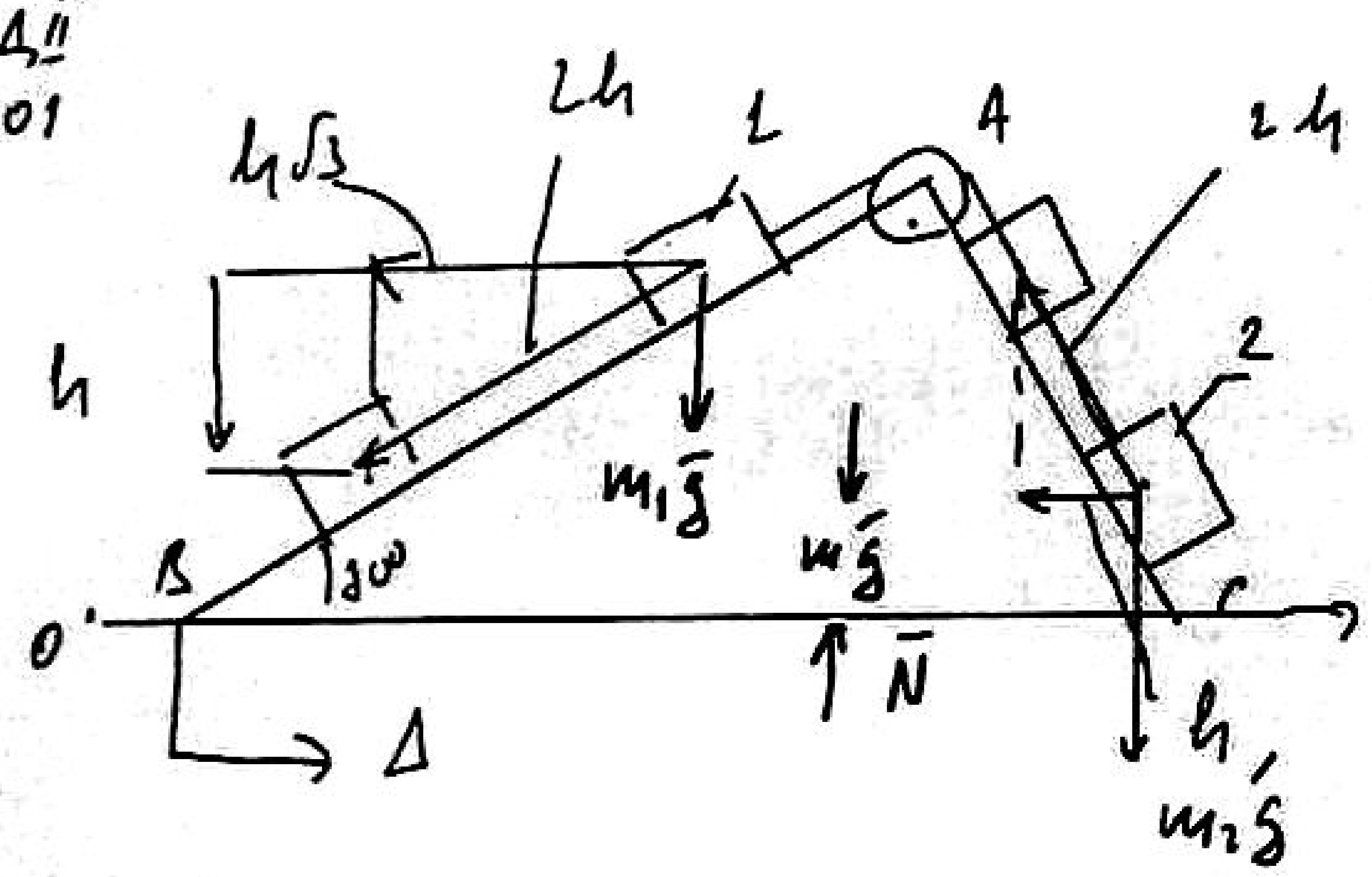
$M\ddot{x}_c = \sum F_{kx}^{(e)} = 0$, $\ddot{x}_c = 0$ $\dot{x}_c = const = \dot{x}_{c0} = 0$

$\boxed{x_c = const}$

$$\Delta x_c = \frac{(m_1+m_2)\Delta - m_2 l \sin\alpha}{m_1+m_2} = 0$$ изменение положения центра масс в проекции на x

$$\boxed{\Delta = \frac{m_2 \frac{l}{2}}{m_1+m_2}}$$ перемещение ползуна при вертикальном положении стержня OA

Д II 01



$m = 4m_1 = 16 m_2$, покой

$h = 10$ см Δ-? перемещение призмы

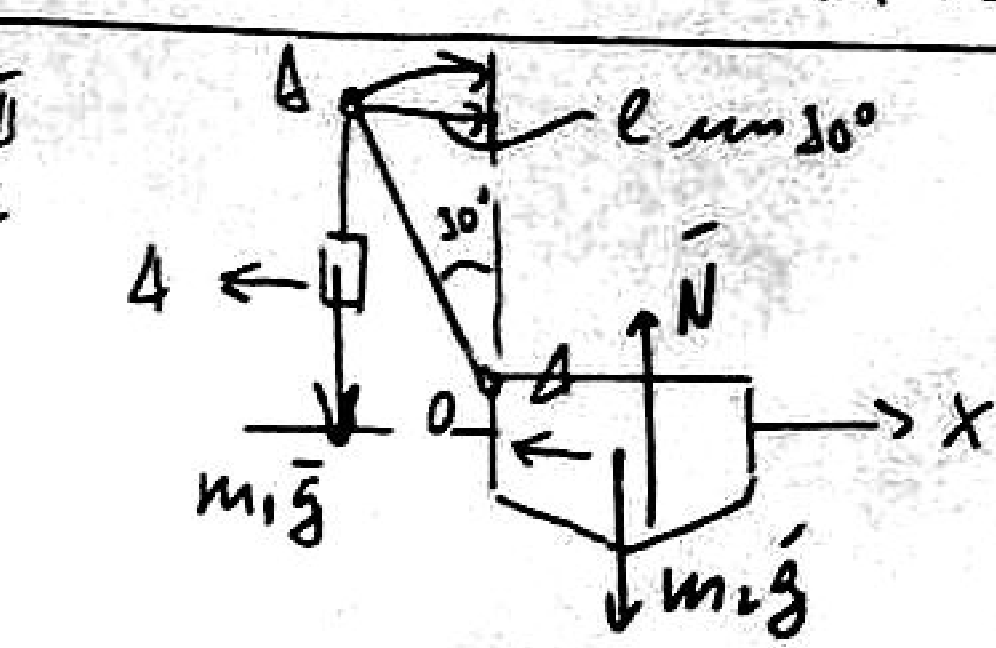
Решение.

$M\ddot{x}_c = \sum F_{kx}^{(e)} = 0$ $\ddot{x}_c = 0$

$\dot{x}_c = const = \dot{x}_{c0} = 0$, $\boxed{x_c = const}$

$$\Delta x_c = 0 = \frac{(m_1+m_2+m)\Delta - m_1 h\sqrt{3} - m_2 h}{m_1+m_2+m}$$

$$\Delta = \frac{m_1\sqrt{3} + m_2}{m+m_1+m_2}\cdot h$$ перемещение призмы



$m_1 = 2T$ $m_2 = 20T$ $OA = l = 8$ м, покой

при $\alpha = 0$ определить перемещение крана!

Решение.

$M\ddot{x}_c = \sum F_{kx}^{(e)} = 0$ $\ddot{x}_c = 0$, $\dot{x}_c = const = \dot{x}_{c0} = 0$

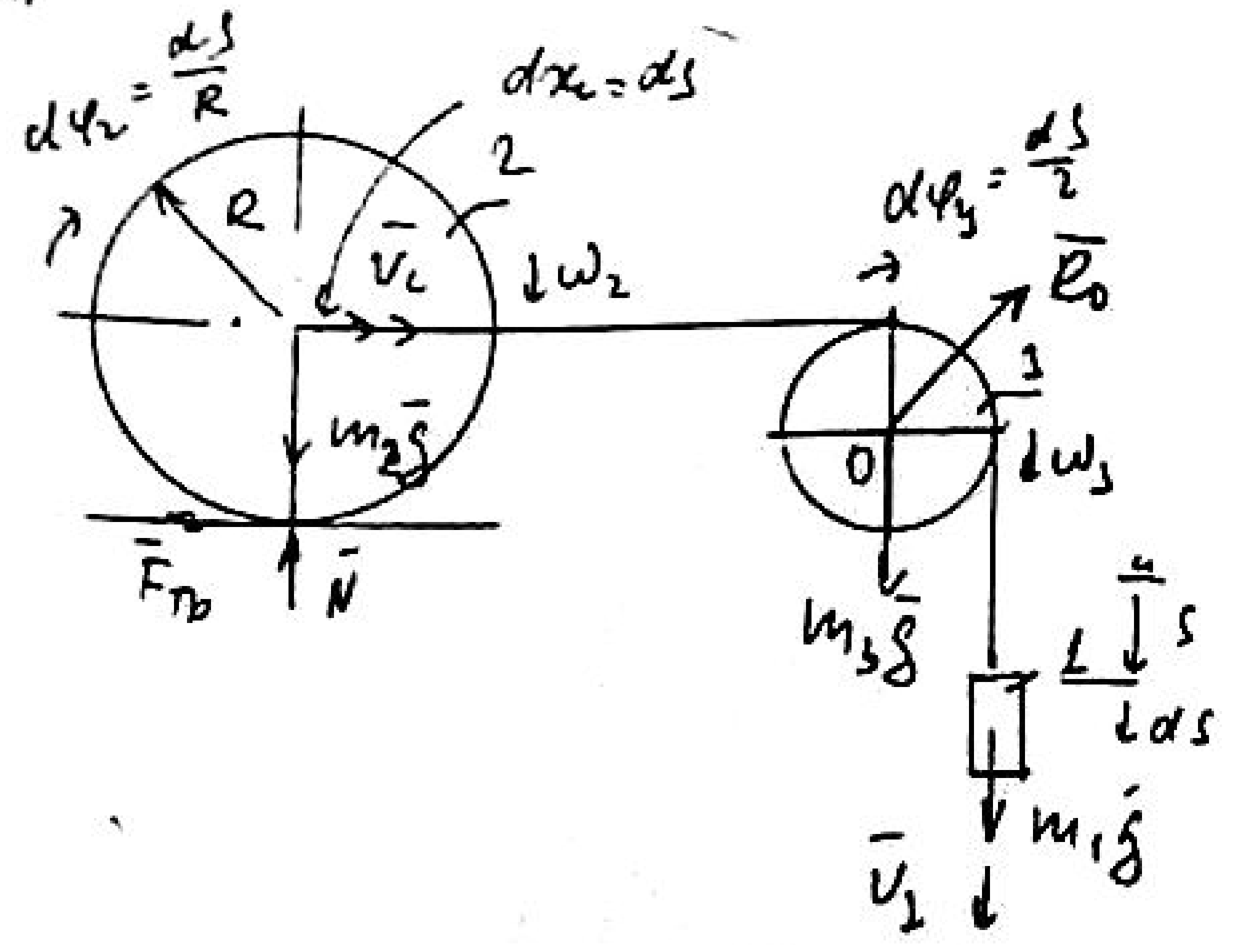
$\boxed{x_c = const}$ $\Delta x_c = 0$

$$\Delta x_c = \frac{-(m_1+m_2)\Delta + m_1 l \sin 30°}{m_1+m_2} = 0$$

$$\boxed{\Delta = \frac{m_1 \frac{l}{2}}{m_1+m_2}}$$

Д II
07-9

13



Дано: m_1, m_2, m_3, R

масса блока 3 — по ободу $T_0=0$

a_1 - ?

$$\boxed{dT = \sum dA(\bar F_k^{(e)}) + \cancel{\sum dA(\bar F_k^{(i)})}}\ 0$$

v_1 - скорость груза 1; $\omega_3 = \frac{v_1}{r}$ угловая скорость блока

$v_C = v_1$; $\omega_2 = \frac{v_C}{R} = \frac{v_1}{R}$ угловая скорость катка

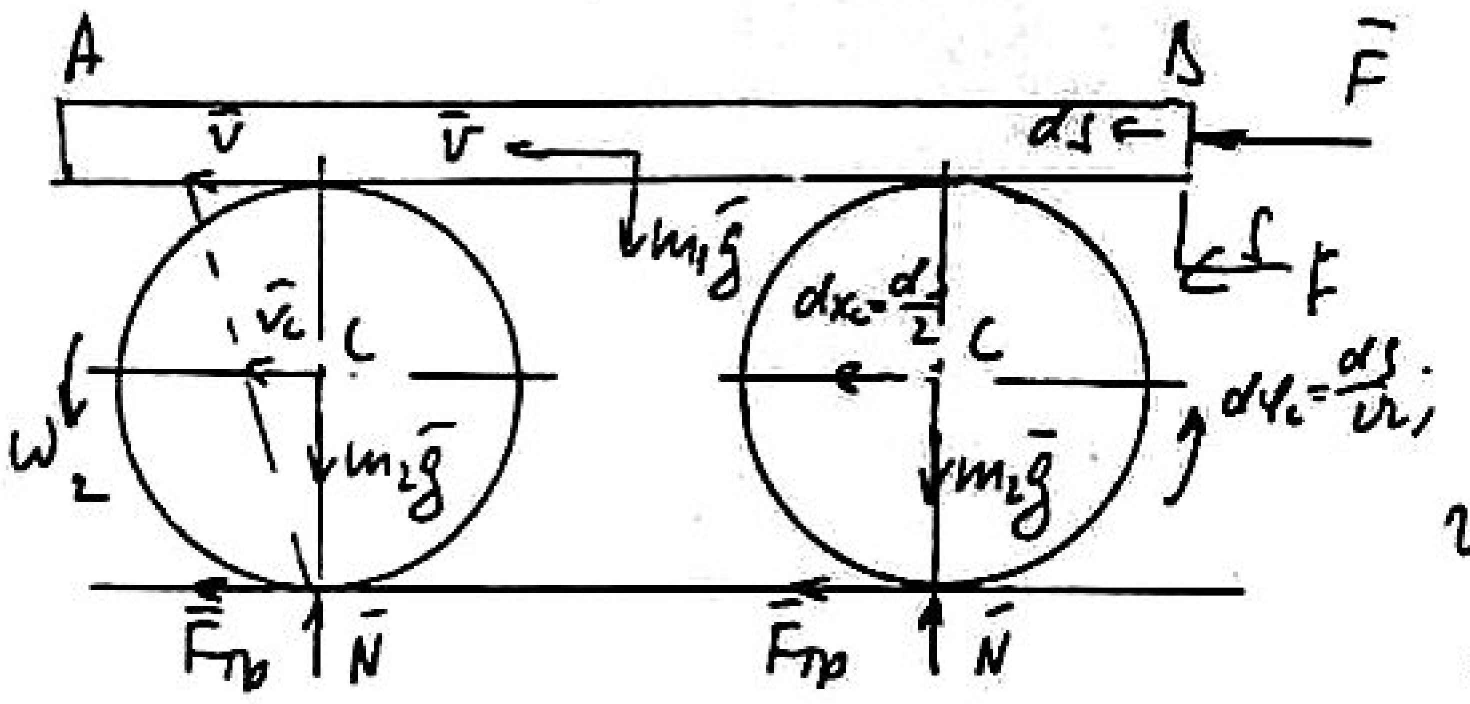
$T_1 = \frac12 m_1 v_1^2$; $T_3 = \frac12 J_{Oz}\omega_3^2 = \frac12 m_3 r^2 \left(\frac{v_1}{r}\right)^2 = \frac12 m_3 v_1^2$

$T_2 = \frac12 m_2 v_C^2 + \frac12 J_{Cz}\omega_2^2 = \frac12 m_2 v_1^2 + \frac12 \frac{m_2R^2}{2}\left(\frac{v_1}{R}\right)^2$; $T=\frac12(m_1+\frac32 m_2+m_3)v_1^2$;

$dT = (m_1+\frac32 m_2+m_3)a_1\,ds$; $\sum dA(\bar F_k^{(e)}) = m_1 g\,ds$;

$(m_1+\frac32 m_2+m_3)a_1 \cancel{ds} = m_1 g \cancel{ds}$; $a_1 = \frac{m_1}{m_1+\frac32 m_2+m_3}g$

Д IV
017



Дано: m_1, m_2, F

ускорение доски?

$$\boxed{dT = \sum dA(\bar F_k^{(e)}) + \sum dA(\bar F_k^{(i)})}$$

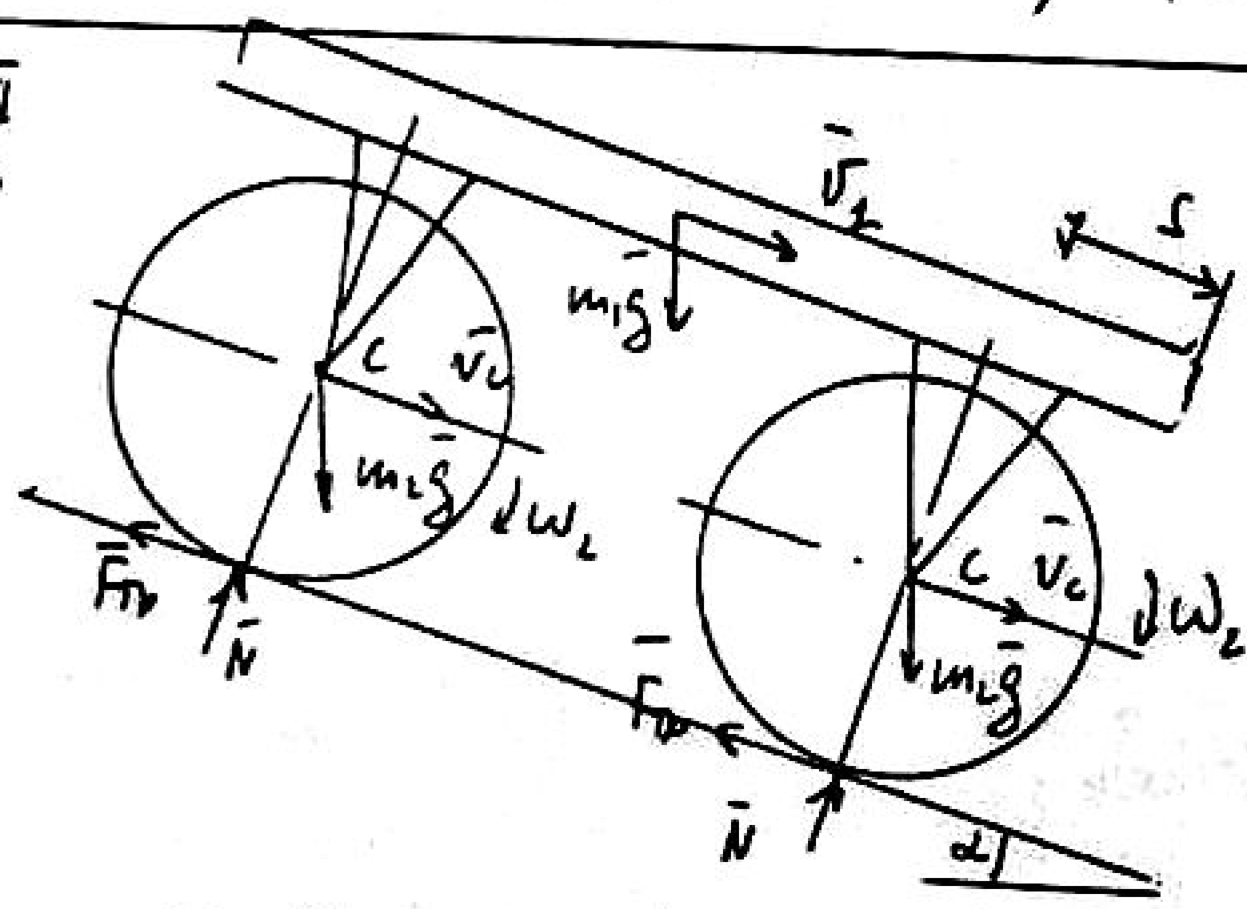
v_1 - скорость доски, $\omega_2 = \frac{v}{2r}$ угловая скорость катка; $v_C = \omega\cdot r = \frac{v}{2}$ скорость центра масс катка

$T_1 = \frac12 m_1 v_1^2$; $T_2 = \frac12 m_2 v_C^2 + \frac12 J_{Cz}\omega_2^2 = \frac12 m_2 \left(\frac{v}{2}\right)^2 + \frac12 \frac{m_2 r^2}{2}\left(\frac{v}{2r}\right)^2$;

$T = T_1 + 2T_2 = \frac12(m_1+\frac34 m_2)v_1^2$; $dT = (m_1+\frac34 m_2)a_1\,ds$

$\sum dA(\bar F_k^{(e)}) = F\,ds$; $(m_1+\frac34 m_2)a_1\,ds = F\,ds$; $\boxed{a_1 = \frac{F}{m_1+\frac34 m_2}}$

Д II
416



Дано: m_1, m_2 - тонкие кольца, α

$v_1 = v_1(s)$, a_1 - ?

$T - \cancel{T_0} = \sum A(\bar F_k^{(e)}) + \cancel{\sum A(\bar F_k^{(i)})}$

v_1 - скорость платформы, $v_C = v$ скорость центра масс колеса, $\omega_2 = \frac{v_C}{r} = \frac{v_1}{r}$ угловая скорость колеса

$T_1 = \frac12 m_1 v_1^2$; $T_2 = \frac12 m_2 v_C^2 + \frac12 J_{Cz}\omega_2^2 = \frac12 m_2 v_1^2 + \frac12\frac{m_2 r^2}{2}\left(\frac{v_1}{r}\right)^2$

$T = T_1 + 4T_2 = \frac12(m_1+6m_2)v_1^2$; $\sum A(\bar F_k^{(e)}) = (m_1+4m_2)g\sin\alpha\cdot s$

$\frac12(m_1+6m_2)v_1^2 = (m_1+4m_2)g\sin\alpha\, s$; $v_1 = \sqrt{\frac{2(m_1+4m_2)g\sin\alpha\, s}{m_1+6m_2}}$;

$dT = \sum dA(\bar F_k^{(e)}) + \sum dA(\bar F_k^{(i)})$, $dT = (m_1+6m_2)a_1\,ds = (m_1+4m_2)g\sin\alpha\,ds = \sum dA(\bar F_k^{(e)})$

$(m_1+6m_2)a_1\cancel{ds} = (m_1+4m_2)g\sin\alpha\cancel{ds}$ $a_1 = \frac{(m_1+4m_2)\sin\alpha}{m_1+6m_2}g$

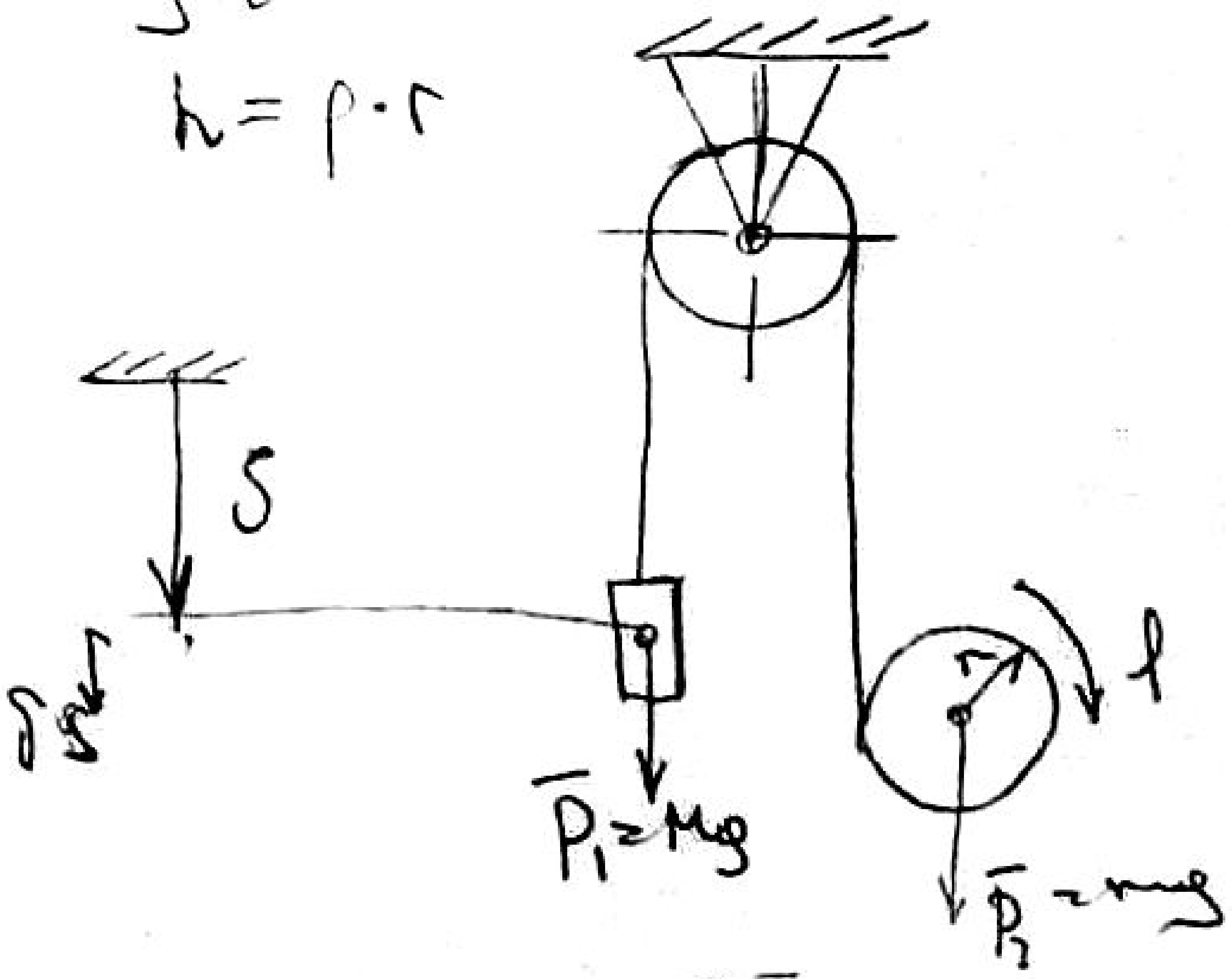
$Q_S = -\frac{F_{упр}\,\delta r}{\delta x} = \ldots\ -cS + mg\sin\alpha$

$Q_\varphi = mg\,\frac{S}{R}$

$\Pi = \cancel{cS}\ \frac{1}{2}cS^2 + Mg\sin\alpha\,S - mg(S + r\varphi)$

mgh

$h = \varphi \cdot r$



$\Pi = MgS - mgS + mgr\varphi$

$Q_S = Mg$

$Q_\varphi = -\ mr$

$\frac{Mg\,\delta S - mg\,\delta S}{\delta S} =$

$= Mg - mg.$

$\delta\varphi \neq 0 \quad \delta S = 0 \quad \frac{mg\,\delta\varphi}{\delta}$

$F_i \frac{\partial r}{\partial \varphi_i}$

$Q_\varphi = F_1 \frac{\partial x}{\partial \varphi} + F_2 \frac{\partial y}{\partial \varphi}$

$x = 2OA\cos\varphi \quad \partial x = -2l\sin\varphi$

$y =$ Ур-е равновесия

$\boxed{\sum_{k=1}^{N} F_k\,\delta r_k = 0}$ $\quad F_{kx}\,\delta x + F_{ky}\,\delta y + M\,\delta\varphi = 0$

$-M\partial\varphi + F\,\delta y + F_1\,\delta x = 0$

$y = OA\sin\varphi \quad x = 2OA\cos\varphi$

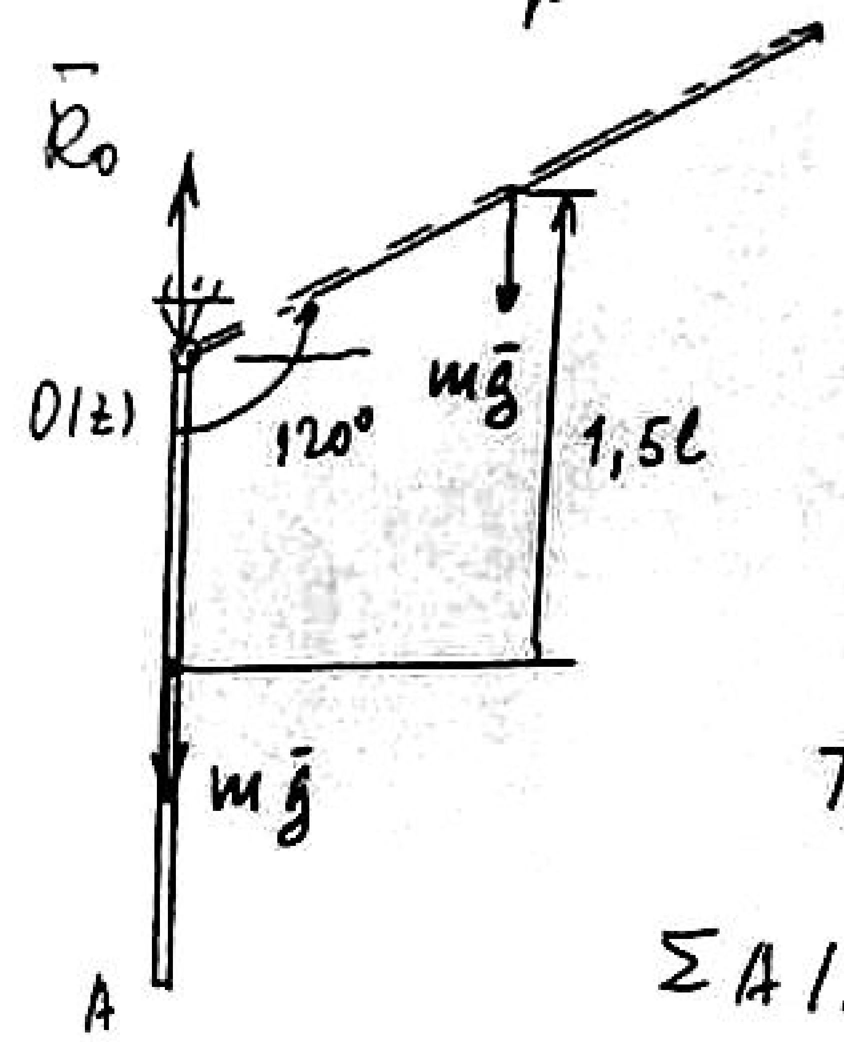
$\delta y = l\cos\varphi\,\delta\varphi \quad \delta x = -2l\sin\varphi\,\delta\varphi$

$-M + Fl\cos\varphi + 2lF_1\sin\varphi = 0$

$-M_A\,\delta\varphi + M\,\delta\varphi + \delta A\,M = 0$

Д. II
433

Теорема об изменении кинетической энергии.



$OA = 2l$, m, угол $120°$ там $\omega = 0$

$\omega_0 - ?$

$$\boxed{T - T_0 = \Sigma A(\bar{F}_k^{(e)}) + \Sigma A(\bar{F}_k^{(i)})}$$

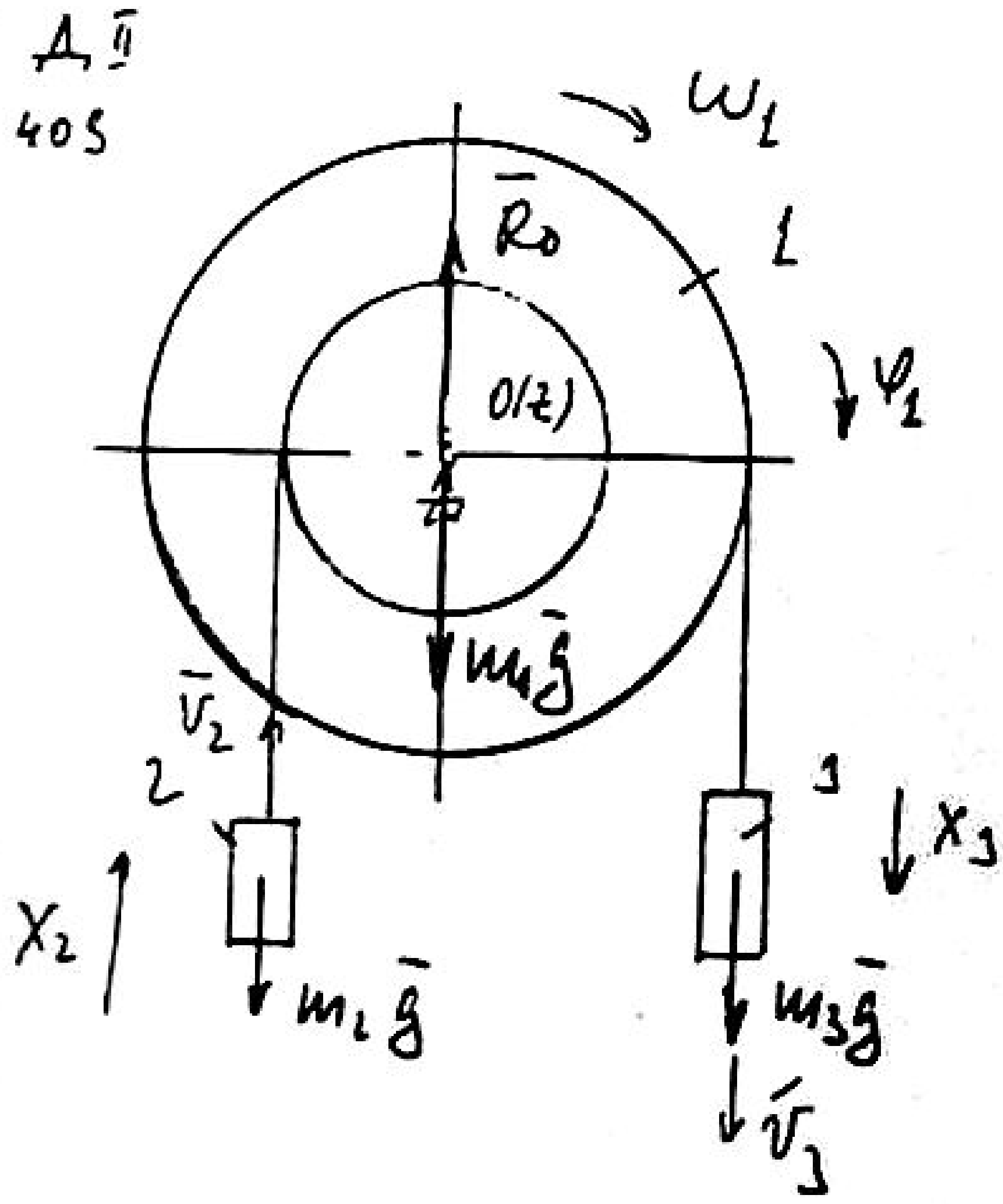
$T = 0$, $\Sigma A(\bar{F}_k^{(i)}) = 0$

$T_0 = \frac{1}{2} J_{Oz} \omega_0^2$, $J_{Oz} = \frac{1}{3} m (2l)^2 = \frac{4}{3} ml^2$, $T_0 = \frac{1}{2} \cdot \frac{4}{3} ml^2\omega$

$\Sigma A(\bar{F}_k^{(e)}) = -mg \cdot 1,5l$

$0 - \frac{2}{3} ml^2 \omega_0^2 = -\frac{3}{2} mgl$, $\omega_0 = \frac{3}{2}\sqrt{\frac{g}{l}}$

Д. II
405



Дано: m_1, R, r, ρ, m_2, m_3

движение из состояния покоя $T_0 = 0$

$\omega_1 = \omega_1(\varphi_1) - ?$

Решение.

ω_1, $v_2 = \omega_1 r$, $v_3 = \omega_1 R$

$x_2 = r\varphi_1$, $x_3 = R\varphi_1$

$T - T_0 = \Sigma A(\bar{F}_k^{(e)}) + \Sigma A(\bar{F}_k^{(i)})$

$T_0 = 0$ $\Sigma A(\bar{F}_k^{(i)}) = 0$

$T = T_1 + T_2 + T_3$, $T_1 = \frac{1}{2} J_{Oz}\omega_1^2 = \frac{1}{2} m_1 \rho^2 \omega_1^2$

$T_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 (r\omega_1)^2$, $T_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2} m_3 (R\omega_1)^2$

$T = \frac{1}{2}(m_1 \rho^2 + m_2 r^2 + m_3 R^2)\omega_1^2$;

$\Sigma A(\bar{F}_k^{(e)}) = m_3 g \cdot x_3 - m_2 g x_2 = m_3 g R\varphi_1 - m_2 g r\varphi_1$

получаем

$\frac{1}{2}(m_1 \rho^2 + m_2 r^2 + m_3 R^2)\omega_1^2 = (m_3 R - m_2 r) g \varphi_1$

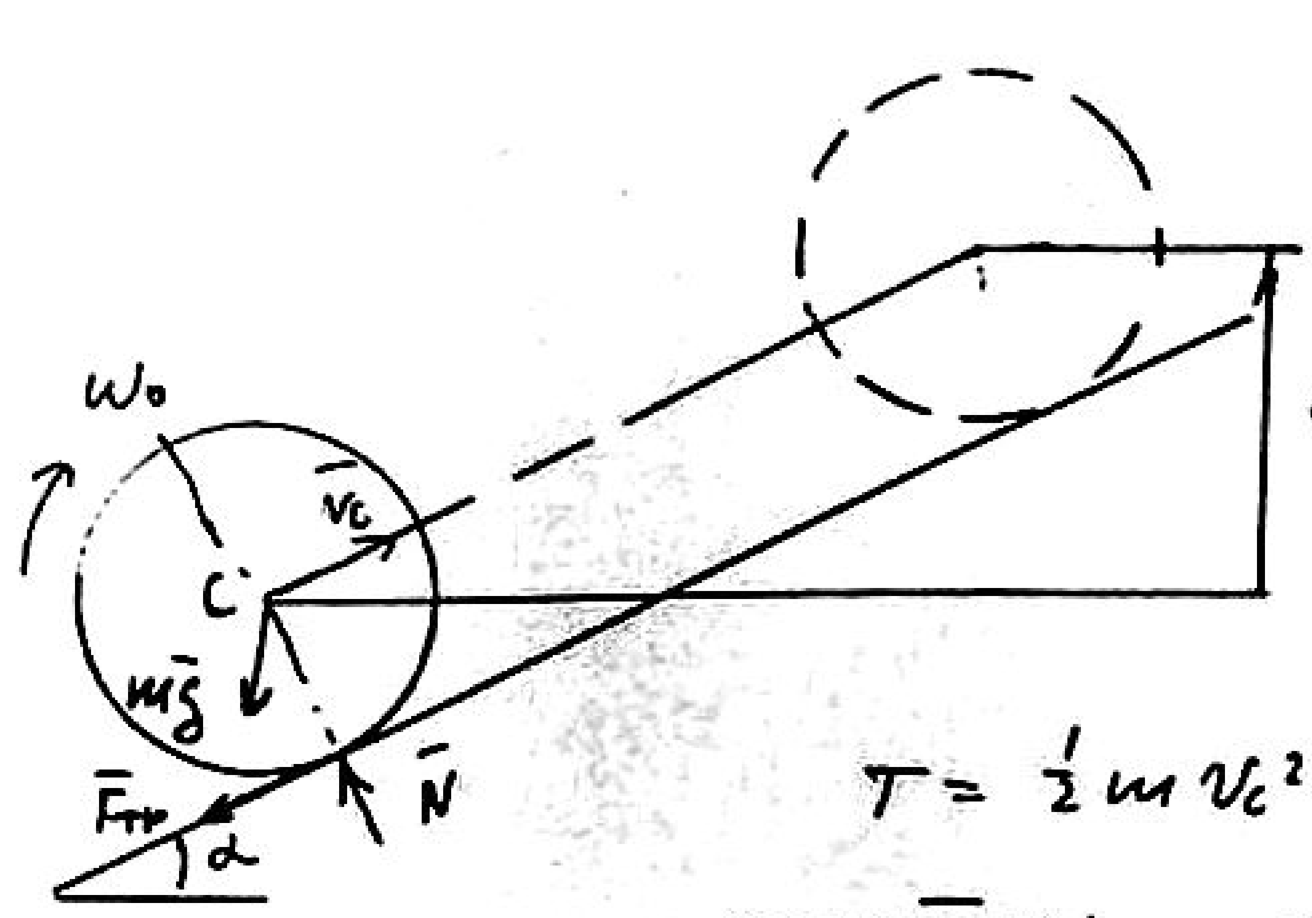
$$\omega_1 = \sqrt{\frac{2(m_3 R - m_2 r) g \varphi_1}{m_1 \rho^2 + m_2 r^2 + m_3 R^2}}$$

+

Теорема об изменении кинетической энергии

(10)

Д II 410



диск радиуса R, ω_0

высота подъема h - ?

диск — плоское движение

$v_C = \omega_0 R$

$$\boxed{T - T_0 = \Sigma A(\bar F_k^{(e)}) + \Sigma A(\bar F_k^{(i)})}$$

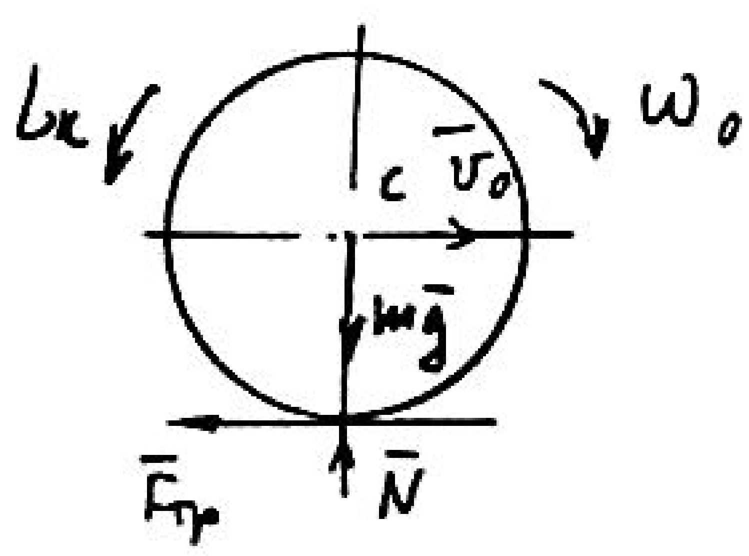
$T = 0$, $\Sigma A(\bar F_k^{(i)}) = 0$

$$T = \frac12 m v_c^2 + \frac12 J_{cz}\omega_0^2 = \frac12 m(\omega_0 R)^2 + \frac12 \frac{mR^2}{2}\omega_0^2 = \frac34 mR^2\omega_0^2$$

$\Sigma A(\bar F_k^{(e)}) = -mgh;$ $\quad 0 - \frac34 \cancel{m} R^2\omega_0^2 = -\cancel{m}gh$

$$h = \frac34 \frac{R^2\omega_0^2}{g}$$

Д II 403



m, r, катится без скольжения

начальная скорость v_0, δ - коэфф. трения качения.

путь до остановки?

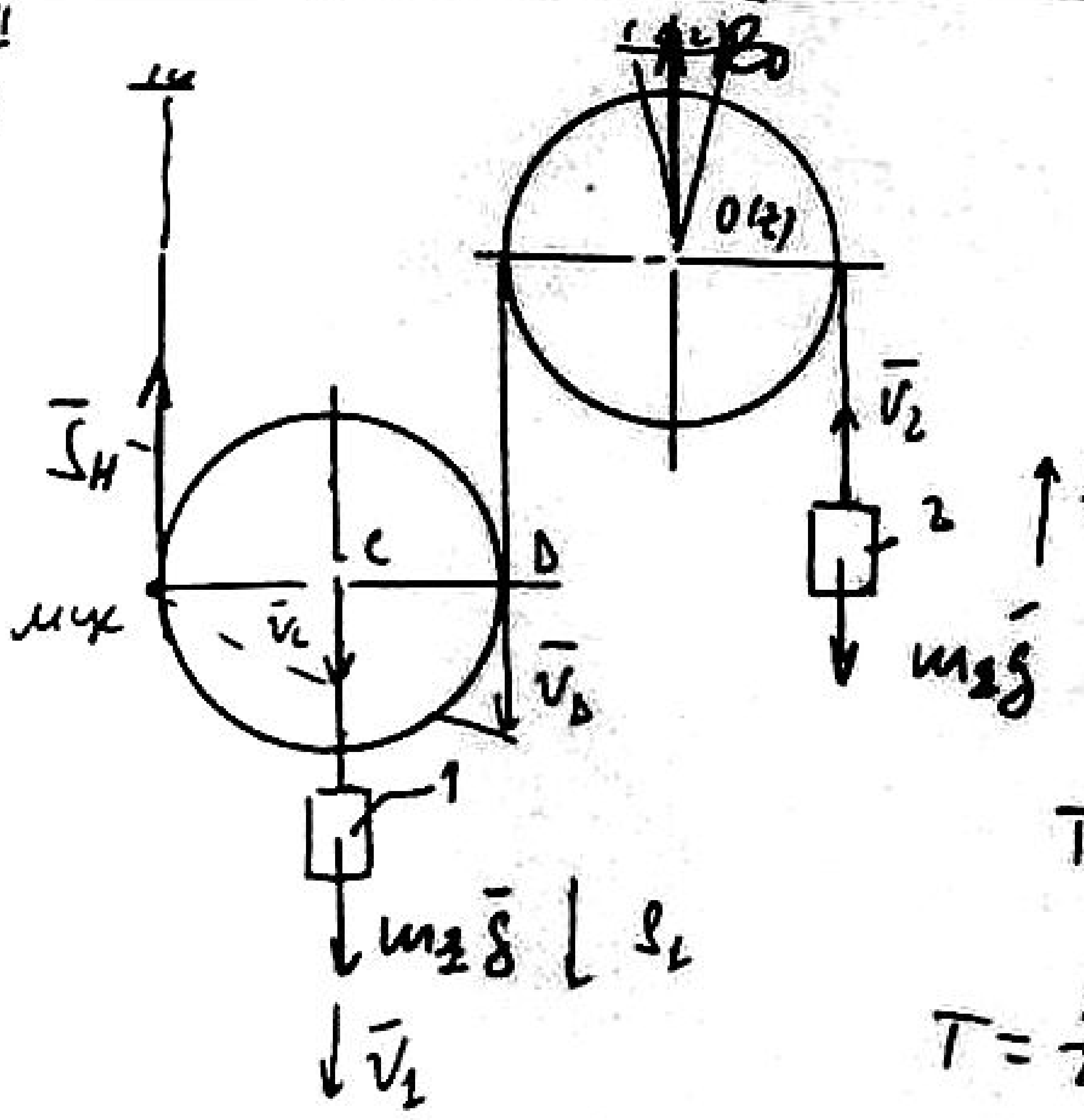
$$\cancel{T}^{\,0} - T_0 = \Sigma A(\bar F_k^{(e)}) + \cancel{\Sigma A(\bar F_k^{(i)})}^{\,0} \quad ; \quad \omega_0 = \frac{v_0}{r}$$

$$T_0 = \frac12 m v_0^2 + \frac12 J_{cz}\omega_0^2 = \frac12 m v_0^2 + \frac12 \frac{m r^2}{2}\left(\frac{v_0}{r}\right)^2 = \frac34 m v_0^2$$

$$\Sigma A(\bar F_k^{(e)}) = -L_k \varphi = -\delta m g \cdot \frac{s}{r};$$

$$0 - \frac34 \cancel{m} v_0^2 = -\delta \cancel{m} g \frac{s}{r}; \qquad s = \frac34 \frac{v_0^2 r}{\delta g}$$

II 8



$m_1 = 8$ кг

$m_2 = 3$ кг

$v_2 = v_2(s_2)$ - ?

$v_1 = v_C$, $\quad v_D = 2v_C = 2v_1 = v_2$

v_2 - скорость груза 2

$v_1 = v_2/2$ скорость груза 1

$$T - \cancel{T_0}^{\,0} = \Sigma A(\bar F_k^{(e)}) + \cancel{\Sigma A(\bar F_k^{(i)})}_{\,0}$$

$$T = \frac12 m_1 v_1^2 + \frac12 m_2 v_2^2 = \frac12 m_1\left(\frac{v_2}{2}\right)^2 + \frac12 m_2 v_2^2 = \left(\frac{m_1}{8} + \frac{m_2}{2}\right) v_2^2$$

$$\Sigma A(\bar F_k^{(e)}) = m_1 g s_1 - m_2 g s_2 \; ; \qquad s_1 = \frac{s_2}{2}$$

$$\Sigma A(\bar F_k^{(e)}) = m_1 g \frac{s_2}{2} - m_2 g s_2$$

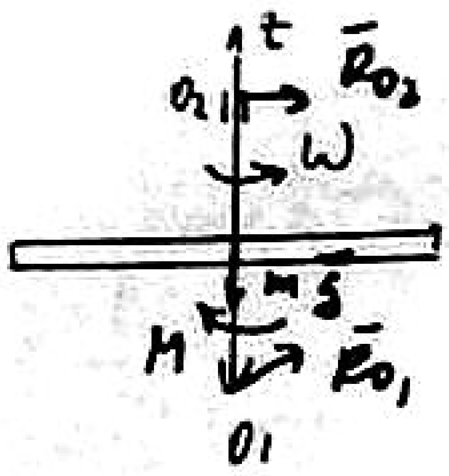
$$\left(\frac{m_1}{8} + \frac{m_2}{2}\right) v_2^2 = \left(\frac{m_1}{2} - m_2\right) g s_2$$

$$v_2 = \frac{m_1/2 - m_2}{\frac{m_1}{8} + \frac{m_2}{2}} g s_2$$

Дифференциальное уравнение вращательного движения

$$J_z \varepsilon = \sum M_z(\bar F_k^{(e)})$$

Д II
505



Дано: m, l, ω_0, торможение $M = \alpha l m \omega$

$\varphi = \varphi(t)$ – ?

$$J_z\ddot\varphi = \sum M_z(\bar F_k^{(e)}) = -M = -\alpha l m\dot\varphi$$

$\frac{ml^2}{12}\cdot\ddot\varphi = -\alpha l m\dot\varphi$, $\boxed{\ddot\varphi + \frac{12\alpha}{l}\dot\varphi = 0}$ $u^2 + \frac{12\alpha}{l}u = 0$

$u_1 = 0$ $u_2 = -\frac{12\alpha}{l}$

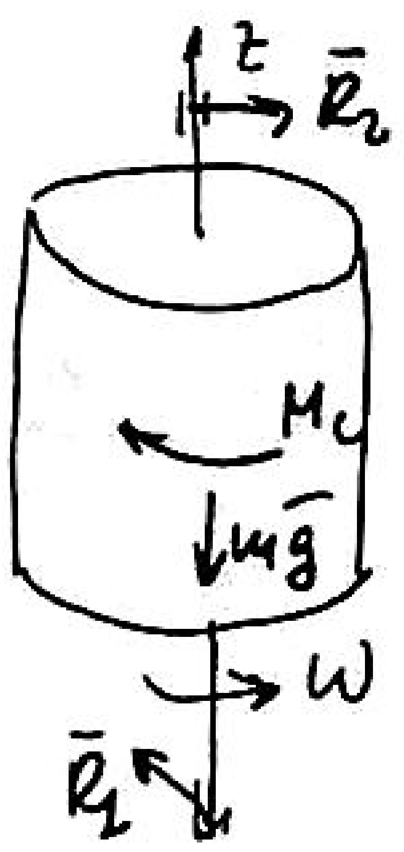
$$\varphi = C_1 + C_2 e^{-\frac{12\alpha}{l}t}$$

$\dot\varphi = -\frac{12\alpha}{l}C_2 e^{-\frac{12\alpha}{l}t}$ н.у. $t=0$ $\varphi=0$ $\dot\varphi=\omega_0$

$0 = C_1 + C_2$, $\omega_0 = -\frac{12\alpha}{l}C_2$, $C_2 = -\frac{\omega_0 l}{12\alpha} = -C_1$

$$\boxed{\varphi = \frac{\omega_0 l}{12\alpha}\left(1 - e^{-\frac{12\alpha}{l}t}\right)}$$

Д II
523



Дано: m, r, ω_0, торможение $M_c = \alpha\omega$

$\omega = \frac{\omega_0}{2}$ время T – ?

$$J\varepsilon = \sum M_z(\bar F_k^{(e)}) = -M_c = -\alpha\omega$$

$$\varepsilon = \frac{d\omega}{dt} = -\frac{\alpha}{J}\omega;$$

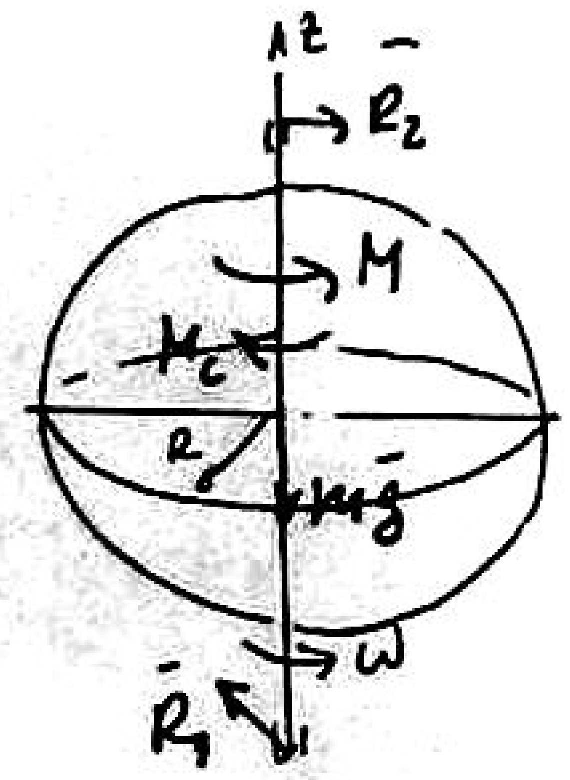
$\frac{d\omega}{\omega} = -\frac{\alpha}{J}dt$, интегрируем

$\ln\omega = -\frac{\alpha}{J}t + C_1$, начальные условия: $t=0$ $\omega=\omega_0$

$\ln\omega_0 = C_1$; $\ln\omega = -\frac{\alpha}{J}t + \ln\omega_0$, $t = \frac{J}{\alpha}\ln\frac{\omega_0}{\omega}$

$t = T$, $\omega = \frac{\omega_0}{2}$; $T = \frac{J}{\alpha}\ln 2$, $J = \frac{mr^2}{2}$, $\boxed{T = \frac{mr^2}{2\alpha}\cdot\ln 2}$

Д II
521



Дано: шар, m, R, $J = \frac{2}{5}mR^2$

$M = const$, $M_c = \alpha\omega$. $t=0$ $\varphi=0$ $\omega=0$

$\varphi = \varphi(t)$ – ?

$$J\ddot\varphi = \sum M_z(\bar F_k^{(e)}) = M - M_c = M - \alpha\dot\varphi$$

$\ddot\varphi + \frac{\alpha}{J}\dot\varphi = \frac{M}{J}$; $\varphi = \varphi_{оо} + \varphi_{чн}$, $u^2 + \frac{\alpha}{J}u = 0$ $u_1 = 0$, $u_2 = -\frac{\alpha}{J}$

$\varphi_{оо} = C_1 + C_2 e^{-\frac{\alpha}{J}t}$, $\varphi_{чн} = Bt$

$0 + \frac{\alpha}{J}B = \frac{M}{J}$, $B = \frac{M}{\alpha}$; $\varphi = C_1 + C_2 e^{-\frac{\alpha}{J}t} + \frac{M}{\alpha}t$ н.у. $t=0$ $\varphi=0$ $\dot\varphi=0$

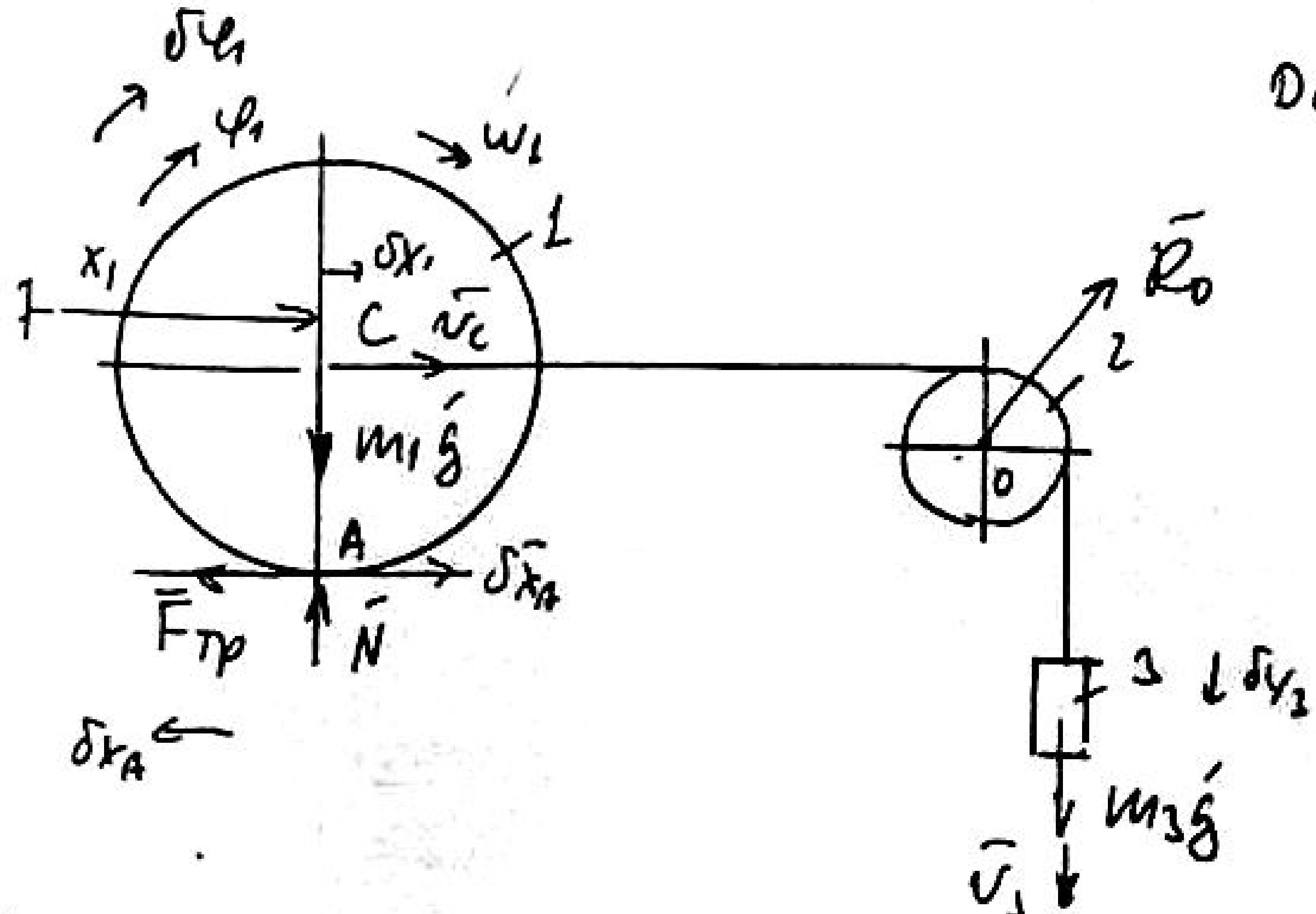
$\dot\varphi = -\frac{\alpha}{J}C_2 e^{-\frac{\alpha}{J}t} + \frac{M}{\alpha}$

$0 = C_1 + C_2$, $0 = -\frac{\alpha}{J}C_2 + \frac{M}{\alpha}$, $C_2 = +\frac{MJ}{\alpha^2} = -C_1$

$$\boxed{\varphi = \frac{MJ}{\alpha^2}\left(e^{-\frac{\alpha}{J}t} - 1\right) + \frac{M}{\alpha}t, \quad J = \frac{2}{5}mR^2}$$

+

Д VI
2-014



Дано: каток, m_1, r_1, f, катится со скольжением
груз m_3
T-? Q_{x_1}, Q_{φ_1} -?

$v_C=\dot x_1$, $\omega_1=\dot\varphi_1$, $v_3=v_C=\dot x_1$

$T_1=\frac12 m_1 v_C^2+\frac12 J_{Cz}\omega_1^2=\frac12 m_1\dot x_1^2+\frac12\frac{m_1r_1^2}{2}\dot\varphi_1^2$; $T_3=\frac12 m_3v_3^2=\frac12 m_3\dot x_1^2$

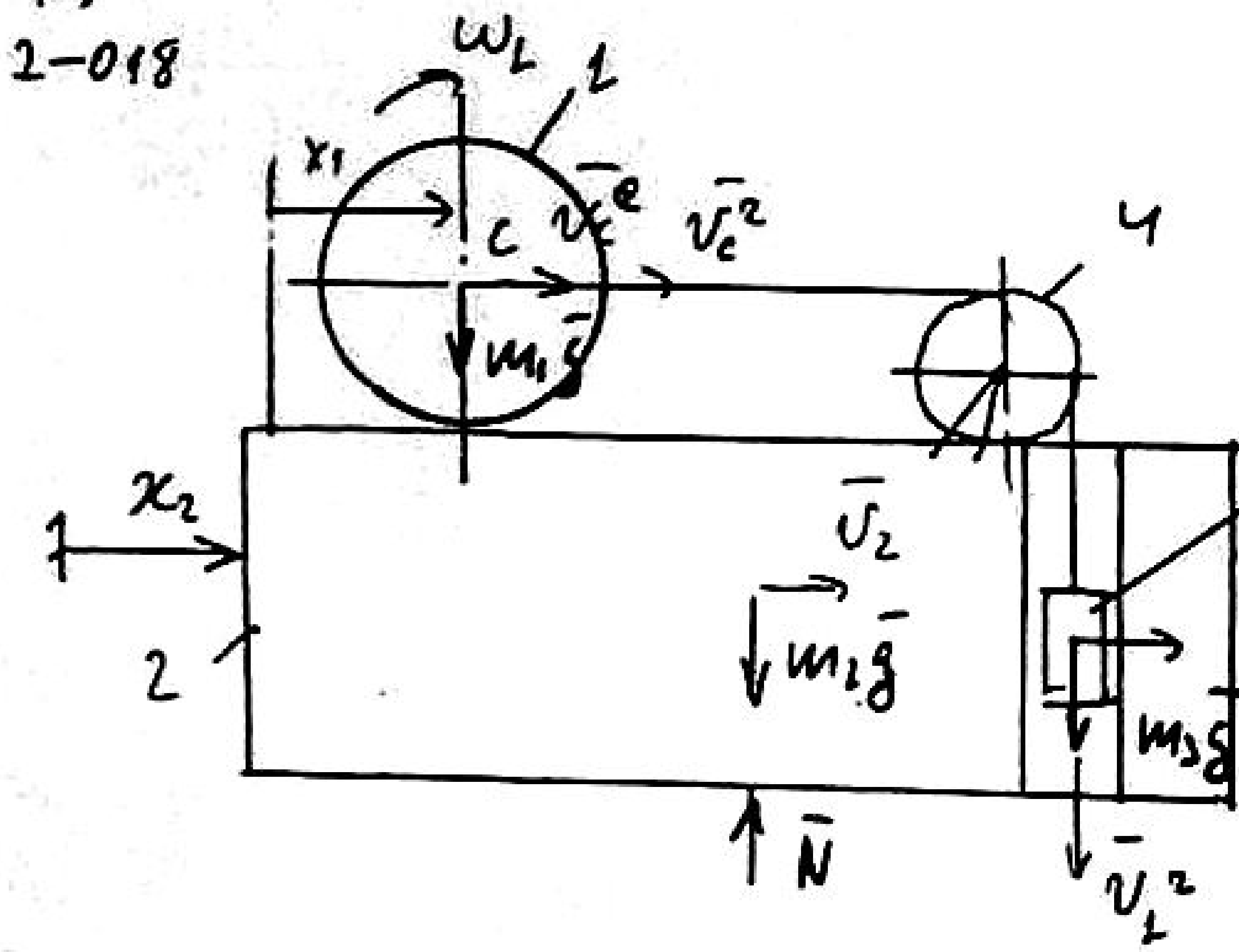
$T=\frac12(m_1+m_3)\dot x_1^2+\frac12\frac{m_1r_1^2}{2}\dot\varphi_1^2$; $Q_{x_1}=\frac{\Sigma[\delta A(\bar F_k)]_{x_1}}{\delta x_1}$; $\delta x_1\neq 0$ $\delta\varphi_1=0$

$\delta x_A=\delta y_3=\delta x_1$ $Q_{x_1}=\frac{-F_{тр}\cdot\cancel{\delta x_A}+m_3g\,\cancel{\delta y_3}}{\cancel{\delta x_1}}=-fm_1g+m_3g$;

$Q_{\varphi_1}=\frac{\Sigma[\delta A(\bar F_k)]_{\varphi_1}}{\delta\varphi_1}$; $\delta\varphi_1\neq 0$ $\delta x_1=0$ $(\delta y_3=0)$ $\delta x_A=r_1\delta\varphi_1$ ← δx_A

$Q_{\varphi_1}=\frac{F_{тр}\cdot\delta x_A}{\delta\varphi_1}=\frac{F_{тр}r_1\delta\varphi_1}{\delta\varphi_1}=F_{тр}r_1=fm_1gr_1$

Д VI
2-018



$v_2=\dot x_2=v_C^e=v_1^e$

$v_C^r=\dot x_1=v_1^r$, $\omega_1=\frac{v_C^r}{r}=\frac{\dot x_1}{r}$

$T_1=\frac12 m_1v_C^2+\frac12 J_{Cz}\omega_1^2=\frac12 m_1(\dot x_1+\dot x_2)^2+\frac12\frac{m_1\cancel{r^2}}{2}\left(\frac{\dot x_1}{\cancel{r}}\right)^2$

$T_2=\frac12 m_2v_2^2=\frac12 m_2\dot x_2^2$

$T_3=\frac12 m_3v_3^2=\frac12 m_3(\dot x_1^2+\dot x_2^2)$

$T=\frac12\left(\frac32 m_1+m_3\right)\dot x_1^2+m_1\dot x_1\dot x_2+\frac12(m_1+m_2+m_3)\dot x_2^2$

Дано: m_1, m_2, m_3
T-? Q_{x_1}, Q_{x_2}-?

$Q_{x_1}=\frac{\Sigma[\delta A(\bar F_k)]_{x_1}}{\delta x_1}$; если $\delta x_1\neq 0$, $\delta x_2=0$

$\delta y_3=\delta x_1$ $Q_{x_1}=\frac{m_3g\cdot\cancel{\delta y_3}}{\cancel{\delta x_1}}=m_3g$

$Q_{x_2}=\frac{\Sigma[\delta A(\bar F_k)]_{x_2}}{\delta x_2}=0$